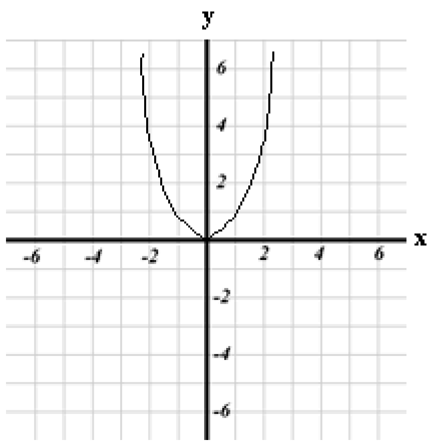
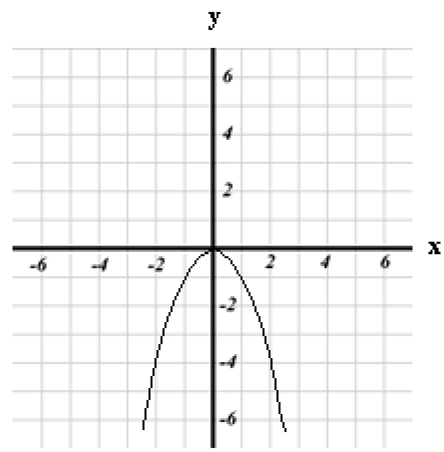
Graphing Quadratic Functions

Recall that any function that can be written in the form of $f(x) = ax^2 + bx + c$, where *a*, *b*, and *c* are real numbers and $a \neq 0$ are quadratic functions. The graph of a quadratic function yields the shape of a parabola. If the value of the "*a*" term is positive the parabola will open upward, whereas if the value is negative it opens downward.



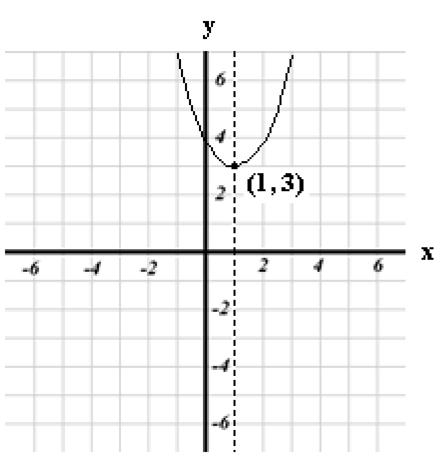
a = "+"



a = "–"

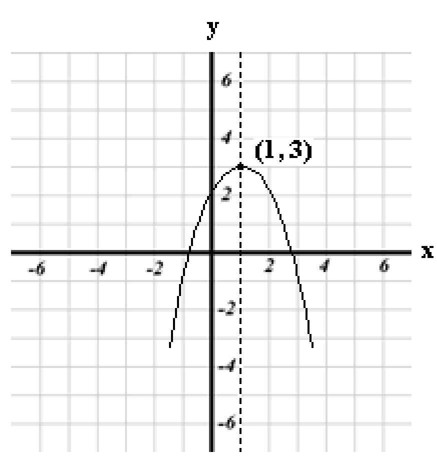
The lowest point of a graph opening upward (the *minimum*), or the highest point of a graph opening downward (the *maximum*) is known as the *vertex*. An *axis of symmetry* can be determined by drawing a vertical line through the vertex of a parabola. This means that the equation for the axis of symmetry will be equal to the *x* value of the vertex.
For example, if the vertex of a parabola was $(1, 3)$, the formula for the axis of symmetry would be $x = 1$. This is without regard to the direction, up or down, that the parabola opens.

Opens upward



x = 1

Opens downward



x = 1

As seen in earlier sections, the process of completing the square is a useful tool in finding non-integer values of quadratic equations, especially intercepts. When a quadratic equation of the form $f(x) = ax^2 + bx + c$ is put through the process of completing the square it yields an equation of the form $f(x) = a(x - h)^2 + k$. The conversion of the equation to this form will yield critical information about the equation's characteristics before you begin to graph it.

1.) The value of h is the distance left (if negative) or right (if positive) the graph translates from the standard position.
2.) The value of k is the distance up (if positive) or down (if negative) the graph translates from the standard position.
3.) The values of h and k, when put together as an ordered pair, give the vertex i.e. (h, k).
4.) The equation $x = h$ is the formula for the axis of symmetry.

The following example demonstrates how to find the following critical information of the equation:

a.) vertex
b.) axis of symmetry
c.) y intercept (if any)
d.) x intercepts (if any)

Example 1: Find the vertex, axis of symmetry, x-intercept(s), and y-intercept and graph the equation $f(x) = x^2 + 2x - 1$.

Solution:

Step 1: y-intercept

In the form $f(x) = ax^2 + bx + c$, $(0, c)$ is the y intercept. In this example, for this step, we need to rewrite the given function to the proper form to get this information.

$f(x) = x^2 + 2x - 1$

$f(x) = x^2 + 2x + (-1)$

Therefore the y-intercept is: (0, -1)

Example 1 (Continued):

Step 2: Complete the square and write in the proper form.

$f(x) = x^2 + 2x - 1$

$f(x) = (x^2 + 2x) - 1$

$f(x) = (x^2 + 2x + 1) - 1 - 1$

$f(x) = (x + 1)^2 - 2$

$f(x) = (x - (-1))^2 + (-2)$

Note: You should put the equation in this form so that you will not make any sign mistakes for the values of h and k .

Step 3: From step 2 the value of h is seen to be –1, while k is equal to –2. The following information can now be determined:

$f(x) = (x - h)^2 + k$

$f(x) = (x - (-1))^2 + (-2)$

Vertex: $(h , k) = (-1, -2)$

Axis of Symmetry: $[x = h] = [x = -1]$

Step 4: Find the x intercepts.

Recall that for all x intercepts, y =0, and that for a function, $f(x) = y$, therefore $f(x) = 0$. Using the results of step 2:

$f(x) = (x + 1)^2 - 2$
$0 = (x + 1)^2 - 2$
$2 = (x + 1)^2$

$\pm\sqrt{2} = \sqrt{(x+1)^2}$

$\pm\sqrt{2} = (x+1)$

$-1\pm\sqrt{2} = x$

Therefore, the x intercepts for this example are:

$\left(-1-\sqrt{2}\ ,\ 0\right)$ and $\left(-1+\sqrt{2}\ ,\ 0\right)$

Example 1 (Continued):

Step 5: Find the "reflection" of the y intercept and graph.

All of the points of a parabola have points that are reflections of each other across the AOS (Axis of Symmetry). Notice that the x-intercepts are a distance of from the x value of –1, the AOS. To find the reflection of the y intercept, duplicate the y value of the point and find the x distance to the AOS then travel the same distance on the other side of the AOS. In this case, the y value of the reflection of the y intercept, (0, -1) is –1, so the reflected point will also have a y value of –1. From the x value of the y intercept, 0 , it is a distance of 1 to the AOS value of –1. The value that is the same distance *on the other side of the AOS* is –2. Therefore, the reflected point of the y intercept is (-2, -1). Using all of this information, plot your points and graph.

Step 6: Graph.