Slope, Modeling, and Linear Relationships

In previous sections, we learned how to write formulas, or equations, in order to represent real-life situations. We call this *mathematical modeling*. We can use mathematical modeling to solve problems in many areas such as science, business, and medicine.

Before we can set up an equation as a mathematical model, we must determine what the variable will be and organize all of the information in terms of that variable.

For example, let's assume that all of the SLAC lab employees will receive a $150 Christmas bonus. If the college must pay $3,300 to cover the cost of the Christmas bonuses, how many employees must be working in the SLAC lab?

We want to know how many employees work in the SLAC lab, so we will set e to be our variable; therefore, we know that e represents the number of employees in the lab. Now, we organize the other information in terms of the variable e. If each employee receives $150, then the total amount of money paid will be $150*e*. (The information given in this problem is simple, but in some cases it will be helpful to set up a table in order to organize the information.)

Now, we can set up the model and solve for the unknown variable.

$$\$3{,}300 = \$150e$$
$$e = 22 \qquad \text{(Divide by \$150.)}$$

There are 22 employees in the SLAC lab. We can check our answer by substituting 22 in for e. We can summarize the steps we used by the following guidelines.

Modeling with Equations:

1. *Determine the variable*. The desired quantity will usually be found in the last sentence of the problem. The notation for this value (*x,a,c,etc.*) can be anything you choose to call it, but it is helpful if it, in some way, represents the desired quantity.
2. *Express all unknown quantities in terms of the variable.* In some cases, it may be helpful to make a table with the information.
3. *Set up the model.* In equation form, organize the expressions listed in Step 2.
4. *Solve the equation and check your answer.* Be sure to express the answer in sentence form, clearly stating the answer to the question in the problem.

The slope of a line tells you the "steepness" of the line (or the rate of change at which the line is increasing or decreasing). The line is increasing if the slope is positive and decreasing if the slope is negative.

In order to find the slope of a line, you must have the coordinates for two points on the line. The slope is then determined by dividing the change in the y-coordinates (referred to as the rise) by the change in the x-coordinates (referred to as the run).

Slope formula: Let (x_1, y_1) and (x_2, y_2) represent two points on a line, where $x_1 \neq x_2$.

$$Slope\ (m) = \frac{y_2 - y_1}{x_2 - x_1}$$

Note: The order in which the coordinates are subtracted does not matter, as long as it is done consistently for both the x and y coordinates. In other words, if you subtract y_1 from y_2 then you must subtract x_1 from x_2.

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{y_1 - y_2}{x_1 - x_2}$$

If the slope is equal to zero then the line is horizontal (parallel to the x-axis). If the slope is undefined (dividing by zero) then the line is vertical (parallel to the y-axis).

Now that you know how to find the slope of a line, you can use the slope when constructing linear models. Lines are also referred to as linear equations, because both variables (x & y) are first-degree variables (variables with an exponent of one).

When finding the equation of a line (constructing a linear model) there are two main formulas that can be used: (1) the slope-intercept form and (2) the point-slope form. The formula that you use will depend upon the information that is given.

Slope-intercept form: $y = mx + b$
Point-Slope form: $y - y_1 = m(x - x_1)$

Slope-Intercept form

The slope-intercept form can be used when you know the slope of the line and the y-intercept. The equation is in the form of $y = mx + b$, where "m" is the slope and "b" is the y-coordinate of the y-intercept.

Point-Slope form

The point-slope form can be used when you know the slope of the line and a point on the line or when you are only given two points on the line. The equation is in the form of $y - y_1 = m(x - x_1)$ where "m" is the slope and (x_1, y_1) are the coordinates of the given point.

Example 1: Lisa and Holly are going to Wisconsin for a cheese convention, and they decide to make the trip in a rental car. The initial cost of renting a car is $20, but there is also an additional charge of 20 cents per mile. From this information:

(a) Create a linear model to represent the cost of the car, *C*, in terms of the distance driven, *d*.

(b) Use the linear model to determine the cost of driving 800 miles.

(c) Draw a graph of the linear model.

Solution:

(a) We are told that the initial cost of the car is $20. At this point Lisa and Holly have not traveled any miles yet (d = 0) so this must be the y-intercept for our linear model.

The additional charge of 20 cents per mile is the rate of change in our cost as the distance traveled increases so this will be the slope for the linear model.

Since we know the slope and y-intercept we would use the slope-intercept form of a line to create our model.

slope (m) = .20
y-intercept (0, b) = (0, 20)
y = mx + b
C(d) = .20d + 20

(b)

C(d) = .20d + 20   d = 800
C(800) = .20(800) + 20
= 160 + 20
= 180

The cost to travel 800 miles is $180.

Example 1 (Continued):

(c)

y-intercept is (0, 20)
C(800) = 180 which is the point (800, 180)