Review Exercise Set 14

Exercise 1: Evaluate the given polynomial when x = -2.

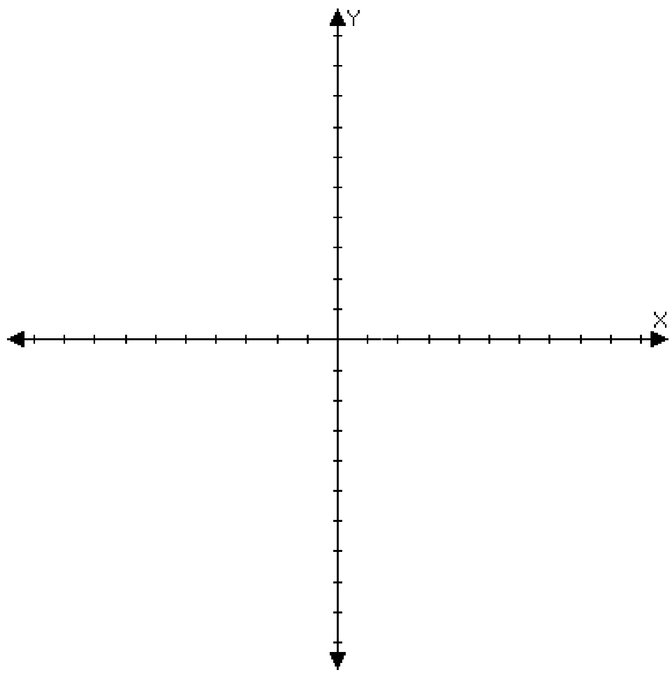
$$P(x) = -2x^3 + x^2 - 3x + 6$$

Exercise 2: For the given parabola determine whether the direction it opens, the y-intercept, the vertex, and x-intercepts (if any).

$$s(x) = -3x^2 + 30x - 72$$

Exercise 3: Use the graph of the base parabola of x and the translation techniques to graph the following parabola.

$f(x) = -(x + 2)^2 - 3$



Exercise 4: Use the completing the square method to express the following parabola in the form of $f(x) = a(x - h)^2 + k$. Identify the vertex.

$f(x) = 2x^2 - 4x + 7$

Exercise 5: Determine the dimensions of a garden that will have the greatest area if a farmer only has 150 feet of fencing with which to enclose it. What would be the maximum area?

Review Exercise Set 14 Answer Key

Exercise 1: Evaluate the given polynomial when x = -2.

$P(x) = -2x^3 + x^2 - 3x + 6$

$P(-2) = -2(-2)^3 + (-2)^2 - 3(-2) + 6$
$P(-2) = -2(-8) + 4 + 6 + 6$
$P(-2) = 16 + 16$
$P(-2) = 32$

Exercise 2: For the given parabola determine whether the direction it opens, the y-intercept, the vertex, and x-intercepts (if any).

$s(x) = -3x^2 + 30x - 72$

a = -3

The parabola opens downward

y-intercept (x = 0)

$s(0) = -3(0)^2 + 30(0) - 72$
$s(0) = 0 + 0 - 72$
$s(0) = -72$

The y-intercept is at (0, -72)

Vertex (a = -3, b = 30)

$$\begin{aligned} x &= -\frac{b}{2a} \\ &= -\frac{30}{2(-3)} \\ &= \frac{-30}{-6} \\ &= 5 \end{aligned}$$

Exercise 2 (Continued):

$$
\begin{aligned}
y &= s\left(-\frac{b}{2a}\right) \\
&= s(5) \\
&= -3(5)^2 + 30(5) - 72 \\
&= -3(25) + 150 - 72 \\
&= -75 + 78 \\
&= 3
\end{aligned}
$$

The vertex is at (5, 3)

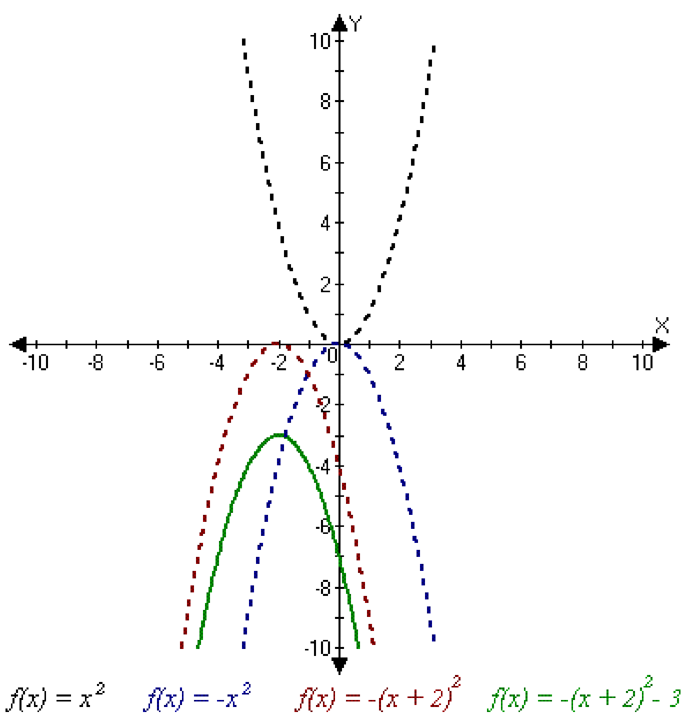
x-intercepts (y = 0)

$0 = -3x^2 + 30x - 72$
$0 = -3(x^2 - 10x + 24)$
$0 = -3(x - 6)(x - 4)$
$x - 6 = 0$ or $x - 4 = 0$
$x = 6$ or $x = 4$

The x-intercepts are at (4, 0) and (6, 0)

Exercise 3: Use the graph of the base parabola of x^2 and the translation techniques to graph the following parabola..

$$f(x) = -(x+2)^2 - 3$$



Exercise 4: Use the completing the square method to express the following parabola in the form of $f(x) = a(x - h)^2 + k$. Identify the vertex.

$$\begin{aligned} f(x) &= 2x^2 - 4x + 7 \\ &= (2x^2 - 4x) + 7 \\ &= 2(x^2 - 2x) + 7 \\ &= 2\left(x^2 - 2x + \left(\frac{-2}{2}\right)^2\right) + 7 - 2\left(\frac{-2}{2}\right)^2 \\ &= 2\left(x^2 - 2x + (-1)^2\right) + 7 - 2(-1)^2 \\ &= 2(x^2 - 2x + 1) + 7 - 2(1) \\ &= 2(x - 1)^2 + 7 - 2 \\ &= 2(x - 1)^2 + 5 \end{aligned}$$

Vertex is (1, 5)

Exercise 5: Determine the dimensions of a garden that will have the greatest area if a farmer only has 150 feet of fencing with which to enclose it. What would be the maximum area?

Use the perimeter formula to express the dimensions in terms of a single variable

$P = 2L + 2W$
$150 = 2L + 2W$
$150 - 2L = 2W$
$75 - L = W$

Substitute the width (in terms of the length) into the Area formula

$A = LW$
$A = L(75 - L)$
$A = 75L - L^2$
$A = -L^2 + 75L$

Exercise 5 (Continued):

Use the formula for the vertex to find the length

$$
\begin{aligned}
L &= -\frac{b}{2a} \\
&= -\frac{75}{2(-1)} \\
&= \frac{-75}{-2} \\
&= 37.5
\end{aligned}
$$

The length would be 37.5 feet.

Find the width

W = 75 - L
W = 75 - 37.5
W = 37.5

The width would be 37.5 feet.

Now, calculate the maximum area

A = LW
A = (37.5 feet)(37.5feet)
A = 1406.25 feet^2

The maximum area of the garden would be 1406.25 square feet.