Variation

If a problem states that there is a functional relationship between two conditions, there is said to exist a variation between them. The type of variation may either be direct or inverse in nature.

The model for direct variation is a linear function of the form $y = k\,x$. This may be read as:

a.) y varies directly as x.
b.) y is directly proportional to k .
c.) $y = k\,x$ for some constant k .

k is known as *the constant of the variation* or *the constant of the proportionality.*

Direct variation problem

Example 1: Hooke's Law for a spring states that the distance a spring is stretched (or compressed) varies directly as the force on the spring. For this problem, a force of 50 lbs. stretches the spring 5 inches.

a.) How far will a force of 20 lbs. stretch the spring?
b.) What force is required to stretch the spring 1.5 inches?

Solution:

Step 1: Determine the formula.

Since the problem states that the distance *(d)* varies directly as the force *(f)* of the spring, the formula would be $d = k\,f$.

Step 2: Substitute the given values into the formula from step 1, and solve for *k*.

The given values of this problem are:

$$d = 5 \text{ inches}, \quad f = 50 \text{ lbs.}$$

Substituted into the problem yields:

$$d = k\,f$$

$$5 = k\,(50)$$

$$\frac{5}{50} = k = \frac{1}{10}$$

The value of k is then substituted with the new conditions given in *a* (step 3) and *b* (step 4) to solve for the required data.

Example 1 (Continued):

Step 3: Find d when *f = 20 lbs.* using the formula found in step 1 and the value of *k* found in step 2.

$$d = k\,f$$

$$d = \left(\frac{1}{10}\right)(20)$$

$$d = 2 \text{ inches}$$

Step 4: Find *f* when *d = 1.5 inches* and *k* = 1/10.

$$d = k\,f$$

$$1.5 = \left(\frac{1}{10}\right)f$$

$$1.5 = 0.1\,f$$

$$\frac{1.5}{.1} = f = 15 \text{ lbs.}$$

Direct variation may also involve relating one variable to a power of another variable. The form of this equation is $y = k\,x^n$ and may be read as:

a.) y varies directly as the N^{th} power of x.
b.) y is directly proportional to the N^{th} power of x.
c.) $y = k\,x^n$ for some constant k .

Direct variation as *N* *th* power

Example 2: The diameter of a particle moved by a stream varies approximately as the square of the velocity of the stream. A stream with the velocity of ¼ mph is able to move sand particles with a diameter of 0.02 inches. What must the velocity of the stream be to move particles of 0.12 inch diameter?

Solution:

Step 1: Determine the formula.

The problem states that the diameter *(d)* of the particle being moved varies approximately as the square of the velocity (v^2). This yields the formula:

$$d = k\,v^2$$

Example 2 (Continued):

Step 2: Substitute the given values into the formula from step 1, and solve for *k*.

The values given in the problem are:

$d = 0.02 \text{ inch}, \qquad v = ¼ \text{ (or } 0.25) \text{ mph}$

These values are substituted into the formula

$$d = k\,v^2$$
$$0.02 = k\,(0.25)^2$$
$$\frac{0.02}{(0.25)^2} = k$$
$$\frac{0.02}{0.0625} = k = 0.32$$

Step 3: The value of *k* from step 2 (0.32) and the new value for *d* given in the problem (0.12) are substituted into the formula found in step 1.

$$d = k\,v^2$$
$$0.12 = (0.32)\,v^2$$
$$\frac{0.12}{0.32} = v^2$$
$$\sqrt{\frac{0.12}{0.32}} = \sqrt{v^2}$$
$$\sqrt{0.375} = v \approx 0.61 \text{ mph}$$

The third type of variation is known as inverse variations. The model for the inverse function is $y = \frac{k}{x}$ and may be written as:

a.) y varies inversely as x.

b.) $y = \frac{k}{x}$ for some constant k.

c.) y is inversely proportional to x.

As with direct variation, the inverse variation may involve an N^{th} power written as $y = \dfrac{k}{x^n}$ and read as:

a.) y varies inversely as the N^{th} power of x.
b.) y is inversely proportional to the N^{th} power of x.

Example 3: A gas law states that the volume of an enclosed gas varies directly to the temperature and inversely as the pressure. The pressure of a gas is 0.75 kg/cm^2 when the temperature is 294° K and the volume is 8000 cc. Find the pressure when the temperature is 300° K and the volume is 7000 cc.

Solution:

Step 1: Determine the formula.

Since the problem states that the volume (v) varies directly as the temperature (t) and inversely to the pressure (p) the formula is:

$$v = \frac{kt}{p}$$

Step 2: Solve for *k* by substituting the values provided by the problem.

The problem provides the following information:

$$p = 0.75 \text{ kg/cm}^2,\ t = 294K,\ v = 8000\ cc$$

for some value of k. These given values are substituted into the formula found in step 1 and k is solved for.

$$v = \frac{kt}{p}$$

$$8000 = \frac{k(294)}{0.75}$$

$$\frac{(8000)(0.75)}{294} = k$$

$$\frac{6000}{294} = k$$

$$\frac{1000}{49} = k$$

Example 3 (Continued):

Step 3: Solve for the new conditions.

The new conditions of the problem are:

t = 300K, *v = 7000 cc*

These new conditions are combined with the value of k determined in step 2 and substituted into the formula found in step 1 and p is solved for.

$$v = \frac{kt}{p}$$

$$7000 = \frac{\left(\frac{1000}{49}\right)(300)}{p}$$

$$p = \frac{\left(\frac{1000}{49}\right)(300)}{\frac{7000}{1}}$$

$$p = \left(\frac{1000}{49}\right)\left(\frac{300}{1}\right)\left(\frac{1}{7000}\right)$$

$$p = 0.87 \ \frac{kg}{cm^2}$$

Joint variation is the term used to describe two different direct variations in the same statement. The form is written as $z = k\,x\,y$, and may be read as:

a.) *z* varies jointly as *x* and *y.*
b.) *z* is jointly proportional to *x* and *y.*
c.) $z = k\,x\,y$ for come constant *k.*

Exponential forms may be used here also such as $z = k\,x^n\,y^m$. This is read as "*z* varies jointly as the N^{th} power of *x* and the M^{th} power of *y.*"

Example 4: The simple interest for a certain savings account is jointly proportional to the time and the principle. After one quarter (3 months) the interest on a principle of $5000 is $106.25. Find the interest after three quarters (9 months).

Solution:

Step 1: Determine the formula.

The problem states that the interest (*i*) is jointly proportional to the time (*t*) and the principle (*p*). This yields the formula:

$$i = k\,p\,t$$

Step 2: Solve for *k* by substituting the values provided by the problem.

The given values of the problem are:

$$i = 106.25, \quad p = 5000 \quad \text{and} \quad t = \tfrac{1}{4}$$

The values are substituted into the formula found in step 1 to solve for *k*.

$$i = k\,p\,t$$
$$106.25 = k\,(5000)\left(\frac{1}{4}\right)$$
$$\frac{(106.25)(4)}{5000} = k = 0.085$$

Step 3: Solve for the new conditions.

The new conditions used to solve for the new interest (*i*):

$$p = 5000 \quad \text{and} \quad t = \frac{3}{4}$$

These values, along with the value of *k* found in step 2, are substituted into the formula found in step 1 to solve for the new interest (*i*).

$$i = k\,p\,t$$
$$i = (0.085)(5000)\frac{3}{4}$$
$$i = \$318.75$$