Slope-Intercept and Point-Slope Forms of a Linear Equation

To determine the equation of a line, you may use two variations of the general form of a line. These formulas are:

1) The *Point-Slope Formula* $(y - y_1) = m(x - x_1)$
2) The *Slope-Intercept Formula* $y = mx + b$

As the names imply the form that you use is dependant on the information you are given to start with.

Example 1: Find the equation of the line that has a slope of $\frac{1}{3}$ and contains the point (2, -1).

Solution

Since the information given is a point and the slope, the point slope formula is used.

Step 1: Substitute the given into the formula.

Since m = $\frac{1}{3}$ and $P_1 = (2, -1)$ then $x_1 = 2$ and $y_1 = -1$.

$y - y_1 = m(x - x_1)$
$y - (-1) = (x - 2)$
$y + 1 = (x - 2)$
$3(y + 1) = 1(x - 2)$
$3y + 3 = x - 2$
$5 = x - 3y$ or $x - 3y = 5$

(This the standard formula of the line)

Step 2: Calculate P_2.

Select any value you with for x or y and substitute it into the equation found in step 1. For this example y will equal 2.

$x - 3y = 5$
$x - 3(2) = 5$
$x - 6 = 6$
$x = 11$

Therefore $P_2 = (11, 2)$

Example 1 (continued):

Step 3: Verify.

When any two points of a line are substituted into the slope formula the slope of the line should be the answer. In this case, when P_1 and P_2 are substituted into the slope formula the answer should be 1/3.

Since $P_1 = (2, -1)$ and $P_2 = (11, 2)$ then $x_1 = 2$, $x_2 = 11$, $y_1 = -1$ and $y_2 = 2$ then:

$$\frac{Y_2 - Y_1}{X_2 - X_1} = \frac{1}{3}$$

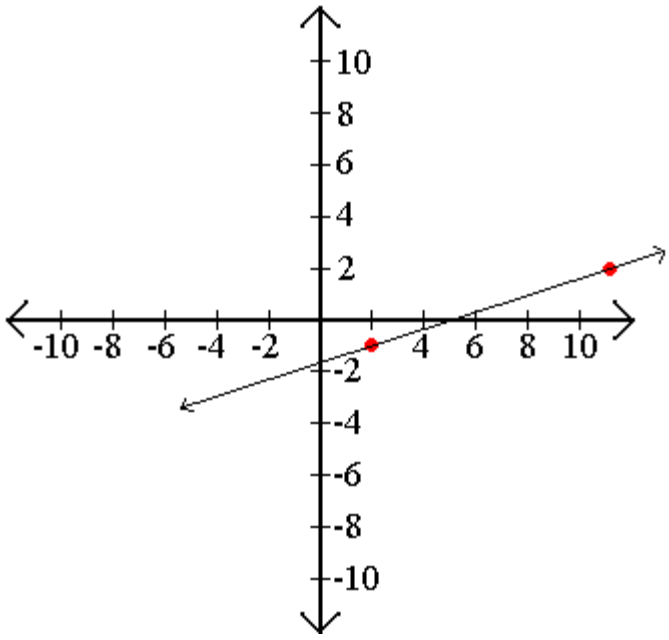
$$\frac{2-(-1)}{11-2} = \frac{1}{3}$$

$$\frac{3}{9} = \frac{1}{3}$$

$$\frac{1}{3} = \frac{1}{3}$$

(The slopes are alike so the equation and P_2 are correct)

Step 4: Graph



The slope intercept formula y = mx + b is used when you know the slope of the line to be examined and the point given is also the y intercept (0, b). In the formula, b represents the y value of the y intercept point.

Example 2: Find the equation of the line that has a slope of 2/3 and a y intercept of (0, 4).

Solution

Step 1: Substitute the given into the formula.

Since the y intercept is (0, 4), b = 4 and the slope, m, is given as 2/3.

$$y = mx + b$$
$$y = \frac{2}{3}x + 4$$
$$\frac{2}{3}x - y = -4$$

(Note: The standard form does not allow fractional values, so you need to resolve this by multiplying by the LCD of 3).

$$3(\frac{2}{3}x - y) = (-4)3$$
$$2x - 3y = -12$$

(This is the calculated equation of the line.)

Step 2: Verify.

Plot 2 points using the formula. For this example $y_1 = 2$ and $y_2 = -6$.

$2x - 3y = -12$	$2x - 3y = -12$
Let $y = 2$	Let $y = -6$
$2x - 3(2) = -12$	$2x - 3(-6) = -12$
$2x - 6 = -12$	$2x + 18 = -12$
$2x = -6$	$2x = -30$
$x = -3$	$x = -15$

Therefore $P_1 = (-3, 2)$ and $P_2 = (-15, -6)$

Example 2 (continued):

Step 2:

Next the x and y values are substituted into the slope formula.

$$M = \frac{Y_2 - Y_1}{X_2 - X_1} = \frac{2}{3}$$

$$\frac{-6-2}{-15-(-3)} = \frac{2}{3}$$

$$\frac{-8}{-12} = \frac{2}{3}$$

$$\frac{2}{3} = \frac{2}{3}$$

Since the slope found using the two points is also $\frac{2}{3}$ the formula is correct

Step 3: Graph