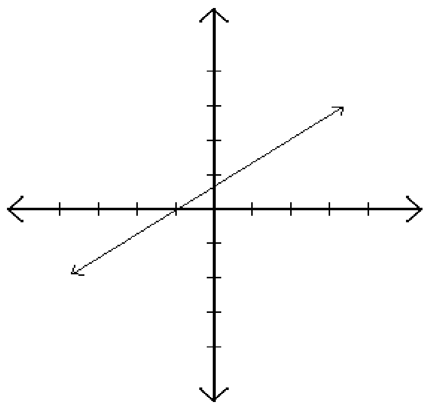
Slope of a Line

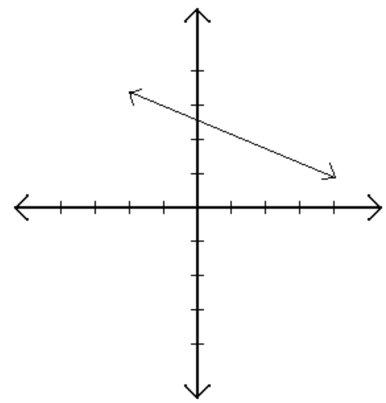
The rate that a given line increases or decreases is called the slope. The following four figures illustrate the only possible types of answers where lines are concerned.

Figure 1



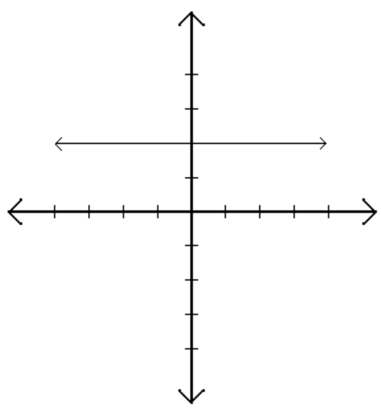
This slope is positive

Figure 2



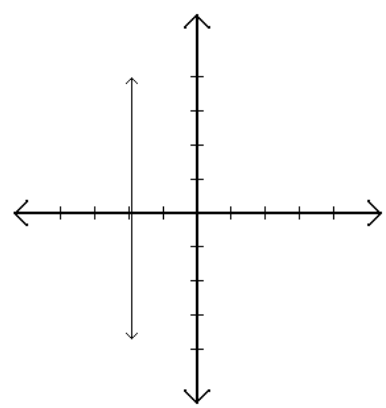
This slope is negative

Figure 1



This slope is zero

Figure 2



This slope is undefined

The formula for the slope of a line is the difference of the y values of two points divided by the difference of their x values:

$$m = \frac{Y_2 - Y_1}{X_2 - X_1}$$

where the points are (x_1, y_1) and (x_2, y_2) and $x_1 \neq x_2$

Example 1: Find the slope and graph the line with the points (-4, 2) and (6, 7).

Solution

Step 1: Establish the X and Y values of the problem.

Let P_1 = (-4, 2) and P_2 = (6, 7)

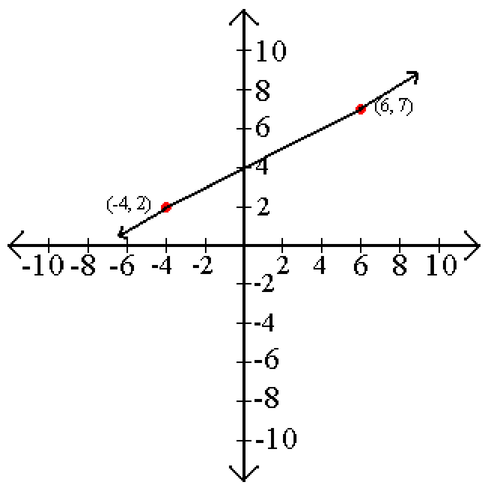
This means that $x_1 = -4$, $x_2 = 6$, $y_1 = 2$, and $y_2 = 7$

Step 2: Substitute these values into the slope formula.

$$m = \frac{Y_2 - Y_1}{X_2 - X_1} = \frac{7-2}{6-(-4)} = \frac{5}{10} = \frac{1}{2}$$

Since the slope is positive the graph should behave like Figure 1.

Step 3: Graph.



Example 2: Find the slope and graph the line with the points (-5, 6) and (3, -2).

Solution

Step 1: Let P_1 = (-5, 6) and P_2 = (3, -2).

This means that $x_1 = -5$, $x_2 = 3$, $y_1 = 6$, and $y_2 = -2$.

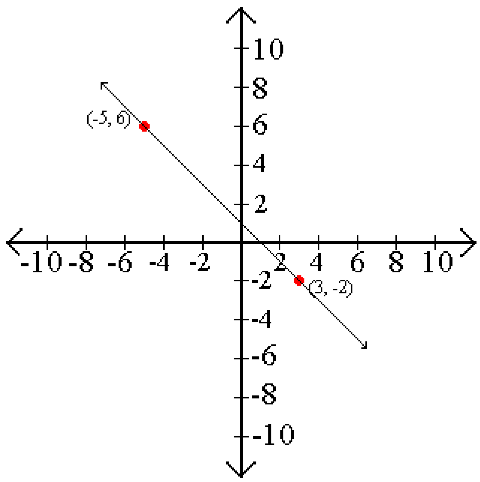
$$m = \frac{-2-6}{3-(-5)} = \frac{-8}{8} = \frac{-1}{1}$$

Example 2 (continued):

Step 2:

Unless otherwise instructed, you should leave the slope in a rational form. Since this slope is negative the graph will behave like Figure 2.

Step 3: Graph.



Example 3: Find the slope and graph the line with the points (-3, 3) and (3, 3).

Solution

Step 1: Let P_1 = (-3, 3) and P_2 = (3, 3).

This means that $x_1 = -3$, $x_2 = 3$, $y_1 = 3$, and $y_2 = 3$.

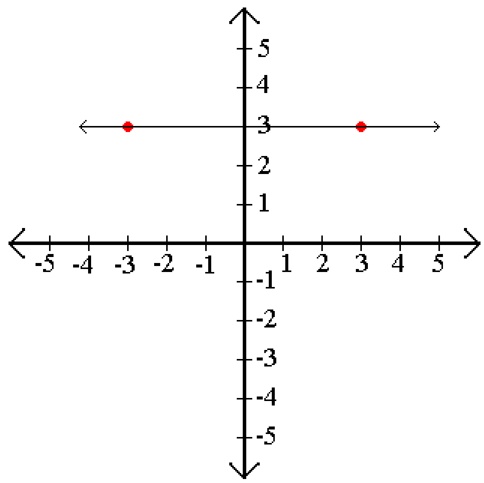
Step 2:

$$m = \frac{3-3}{3-(-3)} = \frac{0}{6} = 0$$

Since the slope is zero the graph will behave like Figure 3.

Example 3 (continued):

Step 3: Graph.



Example 4: Find and graph the slope the line with the points (4, 5) and (4, -3).

Solution

Step 1: Let P_1 = (4, 5) and P_2 = (4, -3).

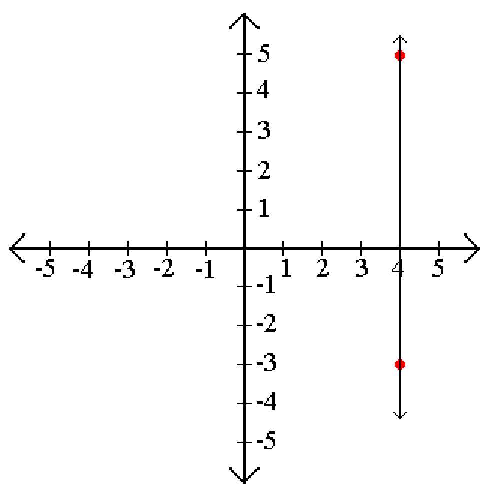
This means that $x_1 = 4$, $x_2 = 4$, $y_1 = 5$, and $y_2 = -3$.

Step 2:

$$m = \frac{-3-5}{4-4} = \frac{-8}{0} = \text{Undefined}$$

Since the slope is undefined the graph will behave like Figure 4.

Step 3: Graph.



Comparing the slopes of two lines can indicate if and how they cross. If two lines do not cross they are said to be parallel and their slopes are the same. If two lines cross forming right (90°) angles they are said to be perpendicular and the product of their slopes equals – 1. If neither of these conditions exists, the lines cross but not at right angles. The next three examples show how to determine these cases.

Example 5: Determine if L_1 and L_2 (line 1 and line 2), are parallel (| |) or perpendicular (⊥) or neither.

$L_1 = 3x + 5y = 10, \qquad L_2 = -6x - 10y = 15$

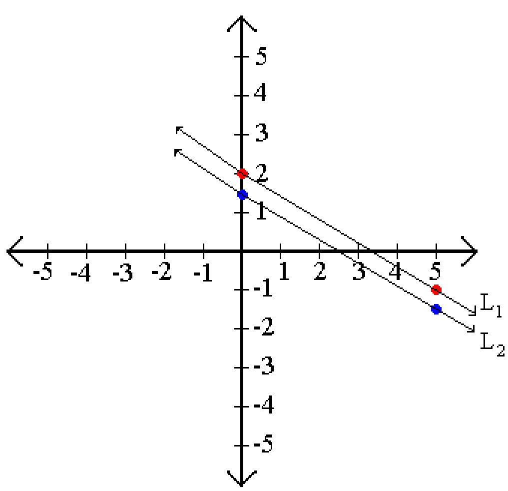
Solution

Step 1: Put L_1 and L_2 in the slope intercept form (y = mx + b) and compare the slopes.

$L_1 = 3x + 5y = 10,$	$L_2 = -6x - 10y = 15$
$5y = -3x + 10$	$-6x - 15 = 10y$
$y = \frac{-3}{5}x + \frac{10}{5}$	$y = \frac{-6}{10}x - \frac{15}{10}$
$M_1 = \frac{-3}{5}$	$M_2 = \frac{-6}{10} = \frac{-3}{5}$

Since $M_1 = M_2$, the lines are parallel.

Step 2: Plot and graph.



Example 6: Determine if L_1 and L_2 (line 1 and line 2), are parallel ($||$) or perpendicular ($\perp$) or neither.

$$L_1 = 4x - y = -7, \qquad L_2 = x + 4y = -12$$

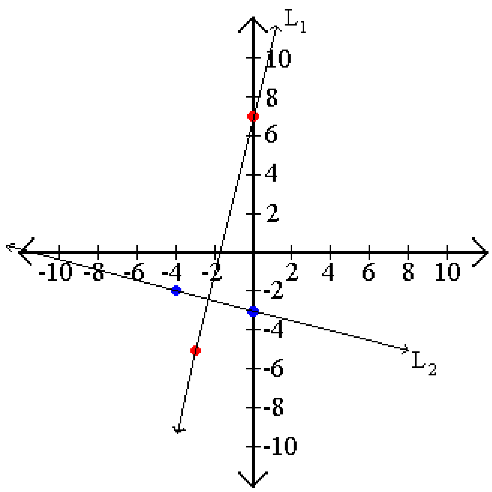
Solution

Step 1: Put L_1 and L_2 into the slope intercept form and calculate the slopes.

$L_1 = 4x - y = -7$	$L_2 = x + 4y = -12$
$4x + 7 = y$	$4y = -x - 12$
$y = 4x + 7$	$y = -x - 3$
$M_1 = 4$	$M_2 = -\frac{1}{4}$

Since $(M_1)(M_2) = (4)(-\frac{1}{4}) = -1$ then the lines are perpendicular.

Step 2: Plot and graph.



Example 7: Determine if L_1 and L_2 (line 1 and line 2), are parallel (| |) or perpendicular ($\perp$) or neither.

$L_1 = 3x + 5y = 7$ $L_2 = -2x + 2y = 5$

Solution

Step 1: Put L_1 and L_2 into the slope intercept form and calculate the slopes.

$L_1 = 3x + 5y = 7$	$L_2 = -2x + 2y = 5$
$5y = -3x + 7$	$2y = 2x + 5$
$y = -\frac{3}{5}X + \frac{7}{5}$	$Y = 1X + \frac{5}{2}$
$M_1 = -\frac{3}{5}$	$M_2 = 1$

Since $M_1 \neq M_2$ and $(M_1)(M_2) \neq -1$, the lines are not parallel or perpendicular, they simply cross.

Step 2: Plot and graph.