Review Exercise Set 19

Exercise 1: Determine the distance between the given pair of points.

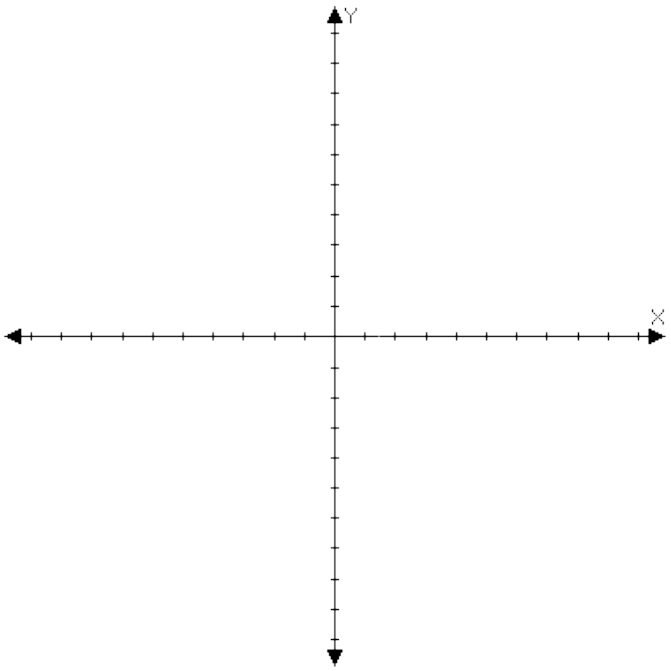
(-2, 6) and (3, -4)

Exercise 2: Determine the midpoint of the line segment between the given pair of points.

(2, 7) and (-3, -1)

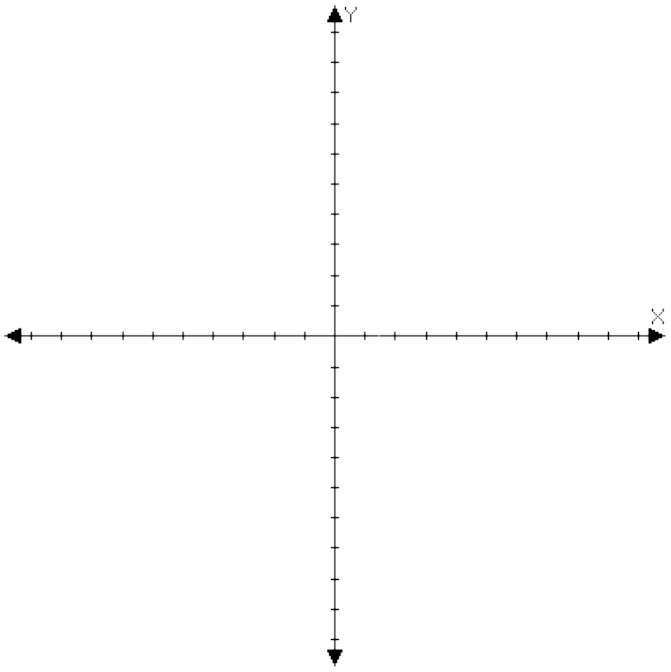
Exercise 3: Find the equation of a parabola with the given vertex and focus. Find the equation of the directrix. Graph the parabola and directrix.

Vertex (0, 0) and Focus (0, -2)



Exercise 4: For the given parabola. Write the equation in the form $y = a(x - h)^2 + k$; identify the direction it opens; find the vertex, intercepts, focus, and equation for the directrix; then graph the parabola.

$y = 2x^2 - 8x + 12$

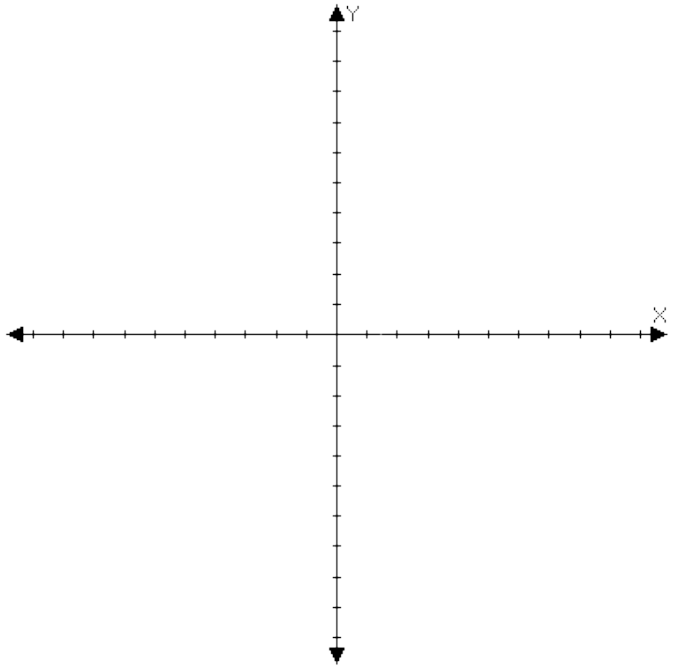


Exercise 5: Write the equation of the circle with the given center and radius.

Center at (3, -4) and Radius of 5

Exercise 6: Write the following equation of a circle in the standard form of $(x - h)^2 + (y - k)^2 = r^2$ and then graph the circle.

$x^2 + y^2 - 8x - 10y + 40 = 0$



Review Exercise Set 19 Answer Key

Exercise 1: Determine the distance between the given pair of points.

(-2, 6) and (3, -4)

$$
\begin{aligned}
d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
&= \sqrt{(-4-6)^2 + (3-(-2))^2} \\
&= \sqrt{(-10)^2 + (5)^2} \\
&= \sqrt{100+25} \\
&= \sqrt{125} \\
&= 5\sqrt{5}
\end{aligned}
$$

Exercise 2: Determine the midpoint of the line segment between the given pair of points.

(2, 7) and (-3, -1)

$$
\begin{aligned}
\text{Midpoint} &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\
&= \left(\frac{2+(-3)}{2}, \frac{7+(-1)}{2} \right) \\
&= \left(-\frac{1}{2}, \frac{6}{2} \right) \\
&= \left(-\frac{1}{2}, 3 \right)
\end{aligned}
$$

Exercise 3: Find the equation of a parabola with the given vertex and focus. Find the equation of the directrix. Graph the parabola and directrix.

Vertex (0, 0) and Focus (0, -2)

From the focus (0, p) we get the value of p

p = -2

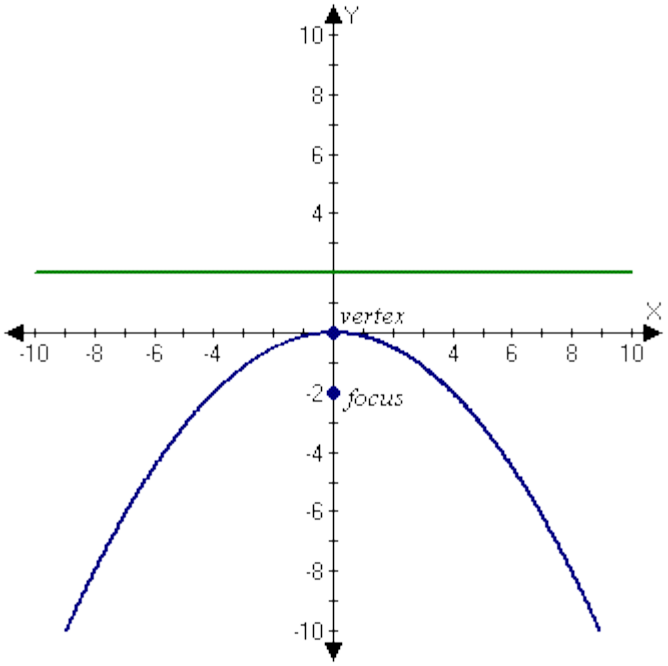
Substitute p into the equation of a parabola with the vertex at the origin

$4py = x^2$
$4(-2)y = x^2$
$-8y = x^2$
$y = -\frac{1}{8} x^2$

Exercise 3 (Continued):

Substitute p into the equation of the directrix

$y = -p$
$y = -(-2)$
$y = 2$



Exercise 4: For the given parabola. write the equation in the form $y = a(x - h)^2 + k$; identify the direction it opens; find the vertex, intercepts, focus, and equation for the directrix; then graph the parabola.

$y = 2x^2 - 8x + 12$

Use the completing the square method to rewrite the equation into the desired form

$y = 2x^2 - 8x + 12$
$y = (2x^2 - 8x) + 12$
$y = 2(x^2 - 4x) + 12$
$y = 2(x^2 - 4x + 4) + 12 - 2(4)$
$y = 2(x - 2)^2 + 12 - 8$
$y = 2(x - 2)^2 + 4$

a = 2 so the parabola opens upward
vertex is at (2, 4)

Exercise 4 (Continued):

Find intercepts

let x = 0
$y = 2(0 - 2)^2 + 4$
$y = 2(-2)^2 + 4$
$y = 2(4) + 4$
$y = 8 + 4$
$y = 12$
y-intercept is at (0, 12)

let y = 0
$0 = 2(x - 2)^2 + 4$
$-4 = 2(x - 2)^2$
$-2 = (x - 2)^2$
$\sqrt{-2} = x - 2$
There are no x-intercepts since the square root of -2 is imaginary

Find focus

$$a = \frac{1}{4p}$$

$$2 = \frac{1}{4p}$$

$$8p = 1$$

$$p = \frac{1}{8}$$

focus is at (h, k + p)

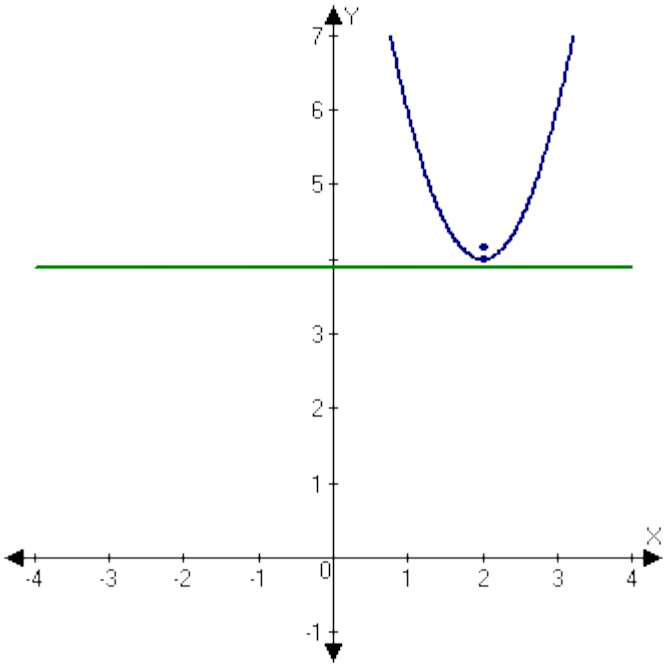
focus = $\left(2, 4 + \frac{1}{8}\right) = \left(2, \frac{32}{8} + \frac{1}{8}\right) = \left(2, \frac{33}{8}\right)$

Find directrix

directrix is at y = k - p

$$y = 4 - \frac{1}{8} = \frac{32}{8} - \frac{1}{8} = \frac{31}{8}$$

Exercise 4 (Continued):



Exercise 5: Write the equation of the circle with the given center and radius.

Center at (3, -4) and Radius of 5

$(x - h)^2 + (y - k)^2 = r^2$
$(x - 3)^2 + (y - (-4))^2 = (5)^2$
$(x - 3)^2 + (y + 4)^2 = 25$

Exercise 6: Write the following equation of a circle in the standard form of $(x - h)^2 + (y - k)^2 = r^2$ and then graph the circle.

$x^2 + y^2 - 8x - 10y + 40 = 0$

Rewrite the equation

$x^2 - 8x + y^2 - 10y = -40$

Use the completing the square method for both variables

$(x^2 - 8x) + (y^2 - 10y) = -40$
$(x^2 - 8x + 16) + (y^2 - 10y + 25) = -40 + 16 + 25$
$(x - 4)^2 + (y - 5)^2 = -40 + 41$
$(x - 4)^2 + (y - 5)^2 = 1$

Exercise 6 (Continued):