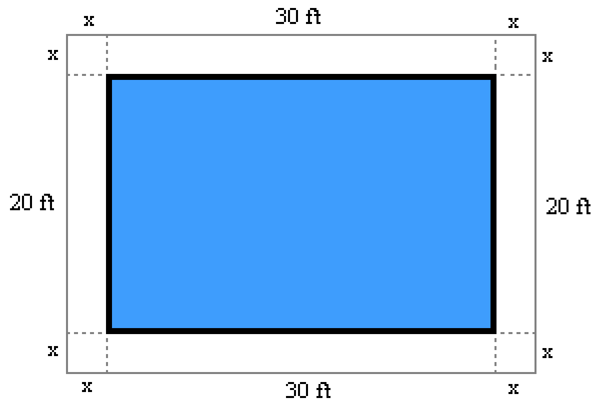
Applications of Quadratic Equations

The following examples show how to approach word problems that involve quadratic equations.

Example 1. Gerald has a swimming pool that is 20 feet by 30 feet. He wants to have a tiled walkway of uniform width around the edge of the pool. If he purchased enough tile to cover 336 square feet how wide will the walkway be?

Solution

Step 1. Draw a diagram.



Step 2. Gather data.

Length of pool and walkway = 2x + 30 ft.
Width of pool and walkway = 2x + 20 ft.
Area formula = $A = L\ X\ W$
Area of pool and walkway = (2x + 30) (2x + 20)
= $4x^2 + 100x + 600$ sq. ft
Length of pool = 30 ft.
Width of pool = 20 ft.
Area of pool = 30 x 20 = 600 sq. ft (A = L x W)
Area of tile = 336 sq. ft. (Given)

Step 3. Set up the equation.

[Area of pool and tile] - [Area of pool] = Area of tile
[$4x^2 + 100x + 600$ sq. ft.] - [600 sq. ft.] = 336 sq ft.
$4x^2 + 100x + 600 – 600 = 336$ sq. ft.
$4x^2 + 100x = 336$ sq. ft.

Example 1 (Continued):

Step 4. Solve for x.

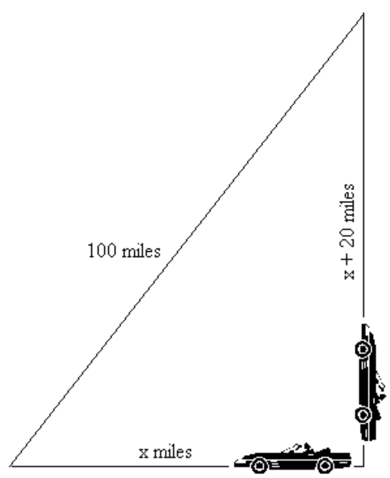
$4x^2 + 100x = 336$
$4x^2 + 100x - 336 = 0$
$4(x^2 + 25x - 84) = 4(0)$
$x^2 + 25x - 84 = 0$
$(x + 28)(x - 3) = 0$
$x + 28 = 0$ or $x - 3 = 0$
$x = -28$ or $x = 3$

Since this is a "real world" problem, the solution cannot be a negative measurement and must therefore be 3 ft.

Example 2. Two cars left an intersection at the same time, one heading due north and the other due west. Some time later they were exactly 100 miles apart. The car heading north had gone 20 miles further than the car heading west. How far had each car traveled?

Solution

Step 1. Draw a diagram.



Step 2. Gather data.

x = Distance traveled by the westbound car.
x + 20 = Distance traveled by the northbound car.
100 = Distance between the cars.

Since the diagram forms a right triangle, the Pythagorean Theorem is used.

Pythagorean Theorem is: $c^2 = a^2 + b^2$

Example 2 (Continued):

Step 3. Substitute values into the equation.

c = 100 , a = x + 20 , b = x therefore:
$c^2 = a^2 + b^2$
$(100)^2 = (x + 20)^2 + (x)^2$
$10{,}000 = x^2 + 40x + 400 + x^2$
$10{,}000 = 2x^2 + 40x + 400$

Step 4. Solve for x.

$2x^2 + 40x + 400 = 10{,}000$
$2x^2 + 40x - 9600 = 0$
$2(x^2 + 20x - 4800) = 2(0)$
$x^2 + 20x - 4800 = 0$
a = 1 , b = 20 and c = -4800

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(20) \pm \sqrt{(20)^2 - 4(1)(-4800)}}{2(1)}$$

$$= \frac{-20 \pm \sqrt{400 + 19200}}{2}$$

$$= \frac{-20 \pm \sqrt{19600}}{2}$$

$$x = \frac{-20 + 140}{2} \quad \text{or} \quad x = \frac{-20 - 140}{2}$$

$$x = \frac{120}{2} \quad \text{or} \quad x = -\frac{160}{2}$$

$$x = 60 \quad \text{or} \quad x = -80$$

Since this is a “real world” problem x is 60 miles.

Therefore:
The distance the westbound car traveled , x, is 60 miles.
The distance the northbound car traveled, x + 20, is 60 +20 = 80 miles.