SOLVING QUADRATIC EQUATIONS BY
THE QUADRATIC FORMULA

Quadratic equations in the form \( ax^2 + bx + c = 0 \) may be solved using the quadratic formula:

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

Example 1. Solve \( 4r^2 = 8r - 1 \) using the quadratic formula.

Step 1. Set the equation equal to zero, determine \( a, b \) and \( c \).

\[ 4r^2 = 8r - 1 \]
\[ 4r^2 - 8r + 1 = 8r - 1 - 8r + 1 \]
\[ 4r^2 - 8r + 1 = 0 \]
\[ a = 4, \ b = -8, \ c = 1 \]

Step 2. Substitute \( a, b, \) and \( c \) into the formula.

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]
\[ x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(4)(1)}}{2(4)} \]

Step 3. Solve for \( x \).

\[ x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(4)(1)}}{2(4)} \]
\[ x = \frac{8 \pm \sqrt{64 - 16}}{8} \]
\[ x = \frac{8 \pm \sqrt{48}}{8} \]
\[ x = \frac{8 \pm 4\sqrt{3}}{8} \]
Step 4  Simplify.

\[
x = \frac{8 \pm 4\sqrt{3}}{8}
\]
\[
x = \frac{4(2 \pm \sqrt{3})}{4(2)}
\]
\[
x = \frac{2 \pm \sqrt{3}}{2}
\]
\[
x = \frac{2 + \sqrt{3}}{2}, \quad \frac{2 - \sqrt{3}}{2}
\]

Example 2.  Solve \(9q^2 + 5 = 6q\) using the quadratic formula.

Step 1.  Set the equation equal to zero, determine \(a, b\) and \(c\).

\[
9q^2 + 5 = 6q
\]
\[
9q^2 + 5 - 6q = 6q - 6q
\]
\[
9q^2 - 6q + 5 = 0
\]
\[
a = 9, \quad b = -6 \text{ and } c = 5
\]

Step 2.  Substitute \(a, b\), and \(c\) into the formula.

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]
\[
x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(9)(5)}}{2(9)}
\]

Step 3.  Solve for \(x\).

\[
x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(9)(5)}}{2(9)}
\]
\[
x = \frac{6 \pm \sqrt{36 - 180}}{18}
\]
\[
x = \frac{6 \pm \sqrt{-144}}{18}
\]
Step 4  Simplify.

\[ x = \frac{6 \pm \sqrt{-144}}{18} \]

\[ x = \frac{6 \pm 12i}{18} \]

\[ x = \frac{6(1 \pm 2i)}{6(3)} \]

\[ x = \frac{1 \pm 2i}{3} = \frac{1 + 2i}{3} \quad \text{or} \quad \frac{1 - 2i}{3} \]

It is possible to predict the nature of the roots you are solving for by using the discriminate of the quadratic equation: \( b^2 - 4ac \).

The three possibilities, based on the solutions are:

1.) If \( b^2 - 4ac < 0 \), then there will be two non-real complex solutions.

2.) If \( b^2 - 4ac = 0 \), then there will be one real solution of multiplicity two.

3.) If \( b^2 - 4ac > 0 \), then there will be two real solutions.

It is possible to check your solutions using the sum of roots or the products of roots. Examples 3 and 4 will use the solutions from examples 1 and 2 to demonstrate this.
Example 3.

Step 1. The solution was $\frac{2\pm\sqrt{3}}{2}$.

Step 2. The sum of roots is:

$$\frac{2 + \sqrt{3}}{2} + \frac{2 - \sqrt{3}}{2} = \frac{2 + \sqrt{3} + 2 - \sqrt{3}}{2} = \frac{4}{2} = 2$$

Step 3. The check is if $2 = \frac{-b}{a}$.

The equation was $4r^2 - 8r + 1 = 0$, therefore $a = 4$ and $b = -8$.

$$\frac{-b}{a} = \frac{-(-8)}{4} = \frac{8}{4} = 2.$$  

Since $2 = 2$ the check works, using the sum of roots.

Step 4. The product of roots is:

$$\left(\frac{2 + \sqrt{3}}{2}\right)\left(\frac{2 - \sqrt{3}}{2}\right) = \frac{4 - 2\sqrt{3} + 2\sqrt{3} - 3}{4} = \frac{1}{4}$$

Step 5. The check is if $\frac{c}{a} = \frac{1}{4}$.

The equation was $4r^2 - 8r + 1 = 0$, therefore $a = 4$ and $c = 1$.

$$\frac{c}{a} = \frac{1}{4}.$$  

Since $\frac{1}{4} = \frac{1}{4}$ the check works, using the product of roots.
Example 4.

Step 1. The solution was \( \frac{1 \pm 2i}{3} \).

Step 2. The sum of roots is:

\[
\frac{1 + 2i}{3} + \frac{1 - 2i}{3} = \frac{1 + 2i + 2 - 2i}{3} = \frac{2}{3}
\]

Step 3. The check is if \( \frac{2}{3} = -\frac{b}{a} \).

The equation was \( 9q^2 - 6q + 5 = 0 \), therefore \( a = 9 \) and \( b = -6 \).

\[
\frac{-b}{a} = \frac{-(6)}{9} = \frac{6}{9} = \frac{2}{3}.
\]

Since \( \frac{2}{3} = \frac{2}{3} \) the check works, using the sum of roots.

Step 4. The product of roots is:

\[
\left( \frac{1 + 2i}{3} \right) \left( \frac{1 - 2i}{3} \right) = \frac{1 + 2i - 2i - 4i^2}{9} = \frac{1 - 4(-1)}{9} = \frac{1 + 4}{9} = \frac{5}{9}
\]

Step 5. The check is if \( \frac{c}{a} = \frac{5}{9} \).

The equation was \( 9q^2 - 6q + 5 = 0 \), therefore \( a = 9 \) and \( c = 5 \).

\[
\frac{c}{a} = \frac{5}{9}.
\]

Since \( \frac{5}{9} = \frac{5}{9} \) the check works, using the product of roots.