SOLVING QUADRATIC EQUATIONS BY THE QUADRATIC FORMULA

Quadratic equations in the form $ax^2 + bx + c = 0$ may be solved using the quadratic formula : $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ **.**

Example 1. Solve $4r^2 = 8r - 1$ using the quadratic formula.

Step 1. Set the equation equal to zero, determine *a, b and c.*

$$4r^2 = 8r - 1$$

$$4r^2 - 8r + 1 = 8r - 1 - 8r + 1$$

$$4r^2 - 8r + 1 = 0$$

$$a = 4\ ,\ b = -8\ ,\ c = 1$$

Step 2. Substitute *a, b,* and *c* into the formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(4)(1)}}{2(4)}$$

Step 3. Solve for x.

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(4)(1)}}{2(4)}$$

$$x = \frac{8 \pm \sqrt{64 - 16}}{8}$$

$$x = \frac{8 \pm \sqrt{48}}{8}$$

$$x = \frac{8 \pm 4\sqrt{3}}{8}$$

Step 4 Simplify.

$$x = \frac{8 \pm 4\sqrt{3}}{8}$$

$$x = \frac{4\left(2 \pm \sqrt{3}\right)}{4(2)}$$

$$x = \frac{2 \pm \sqrt{3}}{2}$$

$$x = \frac{2 + \sqrt{3}}{2}\ ,\ \frac{2 - \sqrt{3}}{2}$$

Example 2. **Solve $9q^2 + 5 = 6q$ using the quadratic formula.**

Step 1. Set the equation equal to zero, determine *a, b and c.*

$9q^2 + 5 = 6q$

$9q^2 + 5 - 6q = 6q - 6q$

$9q^2 - 6q + 5 = 0$

a = 9 , b = -6 and c = 5

Step 2. Substitute *a, b,* and *c* into the formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(9)(5)}}{2(9)}$$

Step 3. Solve for x.

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(9)(5)}}{2(9)}$$

$$x = \frac{6 \pm \sqrt{36 - 180}}{18}$$

$$x = \frac{6 \pm \sqrt{-144}}{18}$$

Step 4 Simplify.

$$x = \frac{6 \pm \sqrt{-144}}{18}$$

$$x = \frac{6 \pm 12i}{18}$$

$$x = \frac{6(1 \pm 2i)}{6(3)}$$

$$x = \frac{1 \pm 2i}{3} = \frac{1+2i}{3} \quad \textbf{or} \quad \frac{1-2i}{3}$$

It is possible to predict the nature of the roots you are solving for by using the discriminate of the quadratic equation: $b^2 - 4ac$.

The three possibilities, based on the solutions are:

1.) **If $b^2 - 4ac < 0$, then there will be two non-real complex solutions.**

2.) **If $b^2 - 4ac = 0$, then there will be one real solution of multiplicity two.**

3.) **If $b^2 - 4ac > 0$, then there will be two real solutions.**

It is possible to check your solutions using the sum of roots or the products of roots. Examples 3 and 4 will use the solutions from examples 1 and 2 to demonstrate this.

Example 3.

Step 1. The solution was $\frac{2 \pm \sqrt{3}}{2}$ **.**

Step 2. The sum of roots is:

$$\frac{2+\sqrt{3}}{2}+\frac{2-\sqrt{3}}{2}=\frac{2+\sqrt{3}+2-\sqrt{3}}{2}=\frac{4}{2}=2$$

Step 3. The check is if 2 = $\frac{-b}{a}$**.**

The equation was $4r^2 - 8r + 1 = 0$**, therefore** ***a = 4 and b = -8.***

$$\frac{-b}{a} = \frac{-(-8)}{4}=\frac{8}{4}=2 \ .$$

Since 2 = 2 the check works, using the sum of roots.

Step 4. The product of roots is:

$$\left(\frac{2+\sqrt{3}}{2}\right)\left(\frac{2-\sqrt{3}}{2}\right)=\frac{4-2\sqrt{3}+2\sqrt{3}-3}{4}=\frac{1}{4}$$

Step 5. The check is if $\frac{c}{a}=\frac{1}{4}$ **.**

The equation was $4r^2 - 8r + 1 = 0$**, therefore** ***a = 4 and c = 1.***

$$\frac{c}{a}=\frac{1}{4} \ .$$

Since $\frac{1}{4}=\frac{1}{4}$ **the check works, using the product of roots.**

Example 4.

Step 1. The solution was $\frac{1 \pm 2i}{3}$ **.**

Step 2. The sum of roots is:

$$\frac{1+2i}{3} + \frac{1-2i}{3} = \frac{1+2i+2-2i}{3} = \frac{2}{3}$$

Step 3. The check is if $\frac{2}{3} = \frac{-b}{a}$**.**

The equation was $9q^2 - 6q + 5 = 0$**, therefore** ***a = 9 and b = -6*****.**

$$\frac{-b}{a} = \frac{-(-6)}{9} = \frac{6}{9} = \frac{2}{3}\ .$$

Since $\frac{2}{3} = \frac{2}{3}$ **the check works, using the sum of roots.**

Step 4. The product of roots is:

$$\left(\frac{1+2i}{3}\right)\left(\frac{1-2i}{3}\right) = \frac{1+2i-2i-4i^2}{9} = \frac{1-4(-1)}{9} = \frac{1+4}{9} = \frac{5}{9}$$

Step 5. The check is if $\frac{c}{a} = \frac{5}{9}$ **.**

The equation was $9q^2 - 6q + 5 = 0$**, therefore** ***a = 9 and c = 5*****.**

$$\frac{c}{a} = \frac{5}{9}\ .$$

Since $\frac{5}{9} = \frac{5}{9}$ **the check works, using the product of roots.**