

Review Exercise Set 3

Exercise 1: The larger of two positive numbers is 2 greater than the smaller. Find the two numbers if their product is 63.

Exercise 2: The length of a rectangle is 4 inches less than twice its width. Find the dimensions if its area is 96 square inches.

Exercise 3: The printable area of a page in a magazine is 70 square inches. The height of a page is twice its width. If the page has a uniform margin of 2 inches all around the page, what are the dimensions of a page in the magazine?

Exercise 4: Larry drove 156 miles in one hour more than it took Sherry to drive 108 miles. Sherry drove at an average rate of 2 miles per hour faster than Larry. How fast did each one travel?

Exercise 5: The perimeter of a rectangle is 44 inches and its area is 112 square inches. Find the length and width of the rectangle if the width is the shorter side.

Review Exercise Set 3 Answer Key

Exercise 1: The larger of two positive numbers is 2 greater than the smaller. Find the two numbers if their product is 63.

Assign variable expressions to the numbers

$$\begin{aligned}x &= \text{smaller positive number} \\x + 2 &= \text{larger positive number}\end{aligned}$$

Setup the equation

We are told that the product of the two numbers is 63 so we would multiply the two variable expressions and set it equal to 63

$$x * (x + 2) = 63$$

Solve for x

$$\begin{aligned}x * (x + 2) &= 63 \\x^2 + 2x &= 63 \\x^2 + 2x - 63 &= 0 \\(x + 9)(x - 7) &= 0\end{aligned}$$

$$\begin{aligned}x + 9 = 0 \text{ or } x - 7 = 0 \\x = -9 \text{ or } x = 7\end{aligned}$$

Since we were told that the numbers had to be positive $x = -9$ cannot be a solution to the equation. Therefore the smaller number (x) must be equal to 7.

Find the larger number $x + 2$

$$\begin{aligned}x + 2 &= 7 + 2 \\x + 2 &= 9\end{aligned}$$

The two numbers are 7 and 9.

Exercise 2: The length of a rectangle is 4 inches less than twice its width. Find the dimensions if its area is 96 square inches.

Assign variable expressions to the dimensions

$$\begin{aligned}x &= \text{width} \\2x - 4 &= \text{length}\end{aligned}$$

Exercise 2 (Continued):

Setup the equation

We are told that the area of the rectangle is 96 so we would multiply the dimensions together and set it equal to 96

$$x * (2x - 4) = 96$$

Solve for x

$$2x^2 - 4x = 96$$

$$2x^2 - 4x - 96 = 0$$

$$2(x^2 - 2x - 48) = 0$$

$$2(x + 6)(x - 8) = 0$$

$$x + 6 = 0 \text{ or } x - 8 = 0$$

$$x = -6 \text{ or } x = 8$$

Since x represents the width of the rectangle it cannot be a negative value. Therefore, the width of the rectangle must be 8.

Find the length

$$2x - 4 = 2(8) - 4$$

$$2x - 4 = 16 - 4$$

$$2x - 4 = 12$$

The dimensions of the rectangle are 8 inches and 12 inches.

Exercise 3: The printable area of a page in a magazine is 70 square inches. The height of a page is twice its width. If the page has a uniform margin of 2 inches all around the page, what are the dimensions of a page in the magazine?

Assign variable expressions to the dimensions

x = width of page

2x = height of page

Since the page has a uniform margin of 2 inches all around the page (top, left, right, bottom), we must subtract 4 inches from each page dimension.

x - 4 = width of printable area

2x - 4 = height of printable area

Exercise 3 (Continued):

Setup the equation

We know the area of the printable area is 70 so we would multiply the dimensions of the printable area together and set it equal to 70

$$(x - 4) * (2x - 4) = 70$$

Solve for x

$$2x^2 - 4x - 8x + 16 = 70$$

$$2x^2 - 12x + 16 = 70$$

$$2x^2 - 12x + 16 - 70 = 0$$

$$2x^2 - 12x - 54 = 0$$

$$2(x^2 - 6x - 27) = 0$$

$$2(x + 3)(x - 9) = 0$$

$$x + 3 = 0 \text{ or } x - 9 = 0$$

$$x = -3 \text{ or } x = 9$$

Since x represents the width of the page it cannot be a negative value. Therefore, the width of the page must be 9.

Find the height of the page

$$2x = 2(9)$$

$$2x = 18$$

The dimensions of the page are 9 inches and 18 inches.

Exercise 4: Larry drove 156 miles in one hour more than it took Sherry to drive 108 miles. Sherry drove at an average rate of 2 miles per hour faster than Larry. How fast did each one travel?

Assign variable expressions to the unknown rates

$$x = \text{Larry's rate}$$

$$x + 2 = \text{Sherry's rate}$$

Exercise 4 (Continued):

Fill in the uniform motion table

	Distance	Rate	Time
Larry	156	x	$\frac{156}{x}$
Sherry	108	$x + 2$	$\frac{108}{x + 2}$

Setup the equation

We are told that Larry drove 1 hour more than Sherry, so Larry's time must equal Sherry's time plus 1.

$$\frac{156}{x} = \frac{108}{x + 2} + 1$$

Solve for x

$$x(x + 2)\frac{156}{x} = x(x + 2)\frac{108}{x + 2} + x(x + 2)1$$

$$(x + 2)(156) = x(108) + x(x + 2)$$

$$156x + 312 = 108x + x^2 + 2x$$

$$156x + 312 = 110x + x^2$$

$$0 = 110x + x^2 - 156x - 312$$

$$0 = x^2 - 46x - 312$$

$$0 = (x + 6)(x - 52)$$

$$x + 6 = 0 \quad \text{or} \quad x - 52 = 0$$

$$x = -6$$

$$x = 52$$

Since x represents the rate at which Larry was traveling it cannot be a negative value. Therefore, the solution is $x = 52$.

Find Sherry's rate

$$x + 2 = 52 + 2$$

$$x + 2 = 54$$

Larry's rate was 52 mph and Sherry's rate was 54 mph.

Exercise 5: The perimeter of a rectangle is 44 inches and its area is 112 square inches. Find the length and width of the rectangle if the width is the shorter side.

Assign variable expressions to the dimensions

x = length of the rectangle

y = width of the rectangle

Since we have two unknown variables we must find a way of expressing the dimensions in terms of a single variable. We are told that the perimeter of the rectangle is 44 inches. We can use the perimeter formula to solve for the width so that both dimensions will be expressed with a common variable.

$$P = 2L + 2W$$

$$44 = 2x + 2y$$

$$44 - 2x = 2y$$

$$22 - x = y$$

x = length of the rectangle

$22 - x$ = width of the rectangle

Setup the equation

Since we know the area of the rectangle, we can now use the formula for the area of a rectangle to setup the equation.

$$LW = A$$

$$(x)(22 - x) = 112$$

Solve for x

$$22x - x^2 = 112$$

$$0 = 112 - 22x + x^2$$

$$x^2 - 22x + 112 = 0$$

$$(x - 8)(x - 14) = 0$$

$$x - 8 = 0 \text{ or } x - 14 = 0$$

$$x = 8 \text{ or } x = 14$$

Since x represents the length of the rectangle and we were told that the width must be the shorter side, the solution for the length would be that $x = 14$.

Find the width

$$22 - x = 22 - 14$$

$$22 - x = 8$$

The dimensions of the rectangle are that the width is 8 inches and the length is 14 inches.