

## Quadratic and Other Inequalities in One Variable

If a quadratic equation is not in the standard form equaling zero, but rather uses an inequality sign ( $<$ ,  $\leq$ ,  $>$ ,  $\geq$ ), the equation is said to be a quadratic inequality. When solving for these types of equations one finds regions in which answers are true instead of single points. The number line is used in the examples to follow to demonstrate this and show how to determine which regions are true or false. The solution sets of these problems are generally written in the interval notation format.

**Example 1:** Solve  $x^2 - x - 12 > 0$ .

### Solution

#### Step 1: Find the critical points.

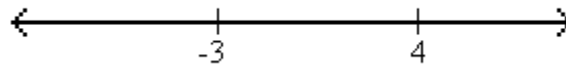
Critical points are those values on the number line which separate the regions of true or false answers and may themselves be either true or false. To find the critical points set the equation equal to zero and solve for the unknown, in this case  $x$ .

$$\begin{aligned}x^2 - x - 12 &= 0 \\(x - 4)(x + 3) &= 0 \\x - 4 = 0 &\quad \text{or} \quad x + 3 = 0 \\x = 4 &\quad \quad \quad \text{or} \quad x = -3\end{aligned}$$

The critical points for this problem are  $-3$  and  $4$ .

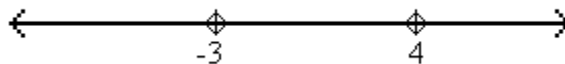
#### Step 2: Construct a diagram of the number line with the critical points.

From Step 1,  $4$  and  $-3$  are seen to be the critical points. A number line is written showing this.



#### Step 3: Determine the nature of the critical points.

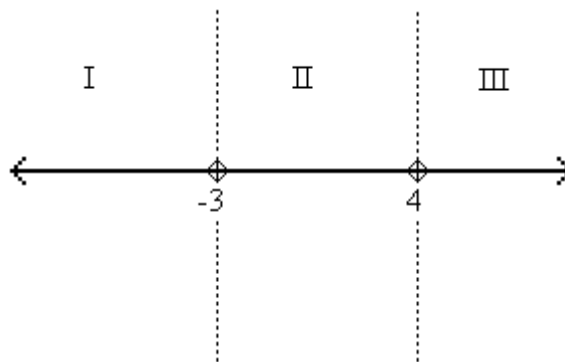
Since the problem states that all the true values will give a result *greater* than zero,  $-3$  and  $4$  are not included in the true solutions because they would give an answer *equal* to zero. This means that on the number line they are shown as open intervals.



**Example 1 (continued):****Step 4: Determine the regional boundaries.**

The regions defined by the problem are as follows:

- a.) Those values less than  $-3$ .
- b.) Those values between  $-3$  and  $4$ .
- c.) Those values greater than  $4$ .



The regions may be labeled in any manner: A, B, C .... 1, 2, 3....I, II, III ... etc. In this case I, II and III are used.

**Step 5: Test values from each region.**

Select a value from within each region (*not the critical points*). Substitute this value into the factors found when solving for the unknown in Step 1. Examine the results of the substitution to determine if the region is positive or negative. For this example the factors were  $(x + 3)$  and  $(x - 4)$ .

**a.) Region I test: -5 is selected**

$$(-5 + 3) = -2 \quad (-5 - 4) = -9 \quad (-2)(-9) = +18$$

Region I is positive.

**b.) Region II test: 0 is selected**

$$(0 + 3) = +3 \quad (0 - 4) = -4 \quad (3)(-4) = -12$$

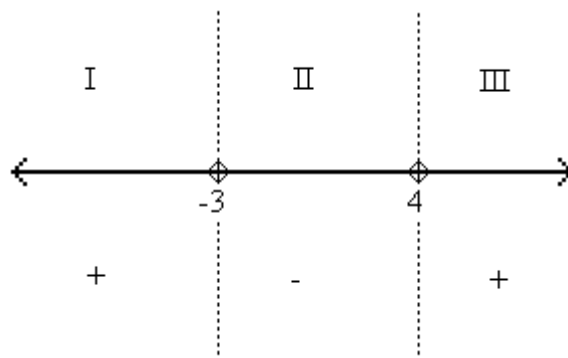
Region II is negative.

**Example 1 (continued):**

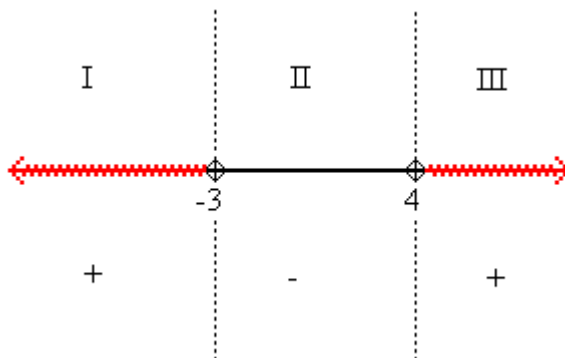
c.) **Region III test: 10 is selected**

$$(10 + 3) = +13 \quad (10 - 4) = +6 \quad (13)(6) = +78$$

Region III is positive.

**Step 6: Graph and solve.**

Since the problem asks for values that make the equation greater than zero, (positive), regions I and III are shaded to indicate that these are the true values.



The solution, in interval notation is:  $(-\infty, -3) \cup (4, +\infty)$ .

**Example 2.** Solve  $\frac{m-2}{m+2} \geq 2$

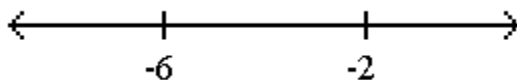
**Solution**

**Step 1: Find the critical points.**

$$\begin{aligned} \frac{m-2}{m+2} &= 2 \\ \frac{m-2}{m+2} - 2 &= 0 \\ \frac{m-2}{m+2} - \left(\frac{2}{1}\right)\left(\frac{m+2}{m+2}\right) &= 0 \\ \frac{m-2-2m-2}{m+2} &= 0 \\ \frac{-m-6}{m+2} &= 0 \\ -m-6 &= 0 \quad \text{or} \quad m+2 = 0 \\ -6 &= m \quad \text{or} \quad m = -2 \end{aligned}$$

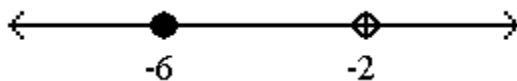
The critical points for this problem are  $-2$  and  $-6$ .

**Step 2: Construct a diagram of the number line with the critical points.**



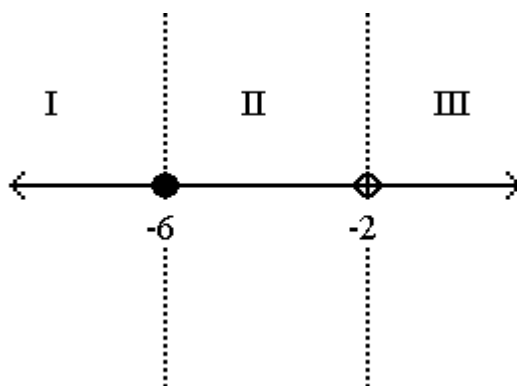
**Step 3: Determine the nature of the critical points.**

Since  $-6$  will make the problem equal to zero if it is substituted into it, it is a closed interval.  $-2$  will be an open interval because it would cause the denominator to be zero if it was substituted into the equation.



**Example 2 (continued):****Step 4: Determine the regional boundaries.**

- a.) Region I are values less than  $-6$ .
- b.) Region II are values between  $-6$  and  $-2$ .
- c.) Region III are values greater than  $-2$ .

**Step 5: Test values from each region.**

The problem to be tested is  $\frac{m-2}{m+2} - 2 \geq 0$

a.) **Region I test: -10 is selected.**

$$\frac{-10-2}{-10+2} - 2 \Rightarrow \frac{-12}{-8} - 2 \Rightarrow \frac{3}{2} - \frac{4}{2} = -\frac{1}{2}$$

The solutions for Region I are negative

b.) **Region II test: -3 is selected.**

$$\frac{-3-2}{-3+2} - 2 \Rightarrow \frac{-5}{-1} - 2 \Rightarrow 5 - 2 = 3$$

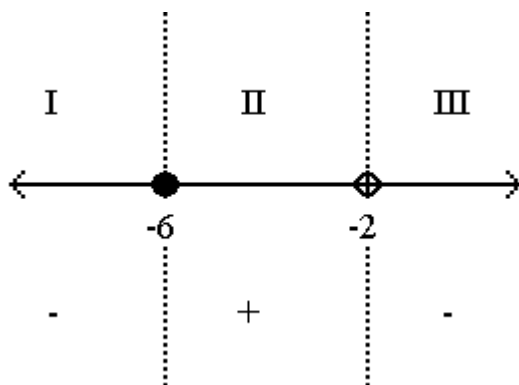
The solutions for Region II are positive.

**Example 2 (continued):**

c.) **Region III test: 10 is selected.**

$$\frac{10-2}{10+2} - 2 \Rightarrow \frac{8}{12} - 2 \Rightarrow \frac{2}{3} - \frac{6}{3} = -\frac{4}{3}$$

The solutions for Region III are negative.

**Step 6: Graph and solve.**

Since the problem asks for values that make the equation greater than or equal to zero, only Region II is shaded. The solution in interval notation is:  $[-6, -2)$

