Systems of Linear Equations: Applications and Problem Solving

Now that we have learned three different methods for solving systems of equations, we can apply these methods to solving real-life situations described in word problems. Earlier when setting up the equation for word problems we had to find a way of describing the information in terms of a single variable but now that is now longer a requirement.

The next few slides provide some examples of how to apply the systems of equations to some common word problem situations.

Example 1: Two cars, one traveling 10 mph faster than the other car, start at the same time from the same point and travel in opposite directions. In 3 hours, they are 300 mile apart. Find the rate of each car.

Solution

Step 1: Assign variable to the unknowns

x = speed of first car
y = speed of second car

Step 2: Setting up a table

Since we are dealing with a distance problem we will setup the table with the column heading of Rate, Time, and Distance.

	Rate	Time	Distance
First Car	x	3	3x
Second Car	y	3	3y

Step 3: Setting up the system of equations

The system of equations will be established using the rate and distance columns. We were told that the rate of one car is 10 mph faster than the other. So we can setup our first equation as:

$$x = y + 10$$

Example 1 (Continued):

Step 3: Setting up the system of equations

Now for the distance column. We were also told that after the 3 hours the distance between the cars is 300 miles. Since the cars are traveling in opposite directions the sum of their individual distances would be equal to the 300 miles. So our second equation would be:

$$3x + 3y = 300$$

Step 4: Solve the system of equations

$$x = y + 10$$
$$3x + 3y = 300$$

Since we already have the first equation solved for x we can use the substitution method to solve this system of equations.

$$3x + 3y = 300$$
$$3(y + 10) + 3y = 300$$
$$3y + 30 + 3y = 300$$
$$6y = 270$$
$$y = 45$$

Now that we know the value for y we can substitute it into the first equation to find x.

$$x = y + 10$$
$$x = 45 + 10$$
$$x = 55$$

The rates of the two cars is 45 mph and 55 mph.

Example 2: A coffee merchant wants to make 6 lb of a blend of coffee costing $5 per pound. The blend is made using a $6-per-pound grade and a $3-per-pound grade of coffee. How many pound of each of these grades should be used?

Solution

Step 1: Assign variable to the unknowns

x = pounds of $6 grade
y = pounds of $3 grade

Example 2 (Continued):

Step 2: Setting up a table

Since we are dealing with a mixture problem we will setup the table with the column heading of Amount, Cost, and Value.

	Amount	Cost	Value
\$6 grade	x	6	6x
\$3 grade	y	3	3y

Step 3: Setting up the system of equations

The system of equations will be established using the amount and value columns. We were told that the total amount of coffee for the blend is to be 6 pounds. So the sum of the amounts of the two blends must equal 6.

$$x + y = 6$$

Now for the value column. We were also told that coffee blend would be 6 pound with a cost of \$5 per pound. So the value of the blend would be \$30 (6 pounds × \$5 per pound). The sum of the values of the two individual coffees used in the blend must equal the total value of the blend. So our second equation would be:

$$6x + 3y = 30$$

Step 4: Solve the system of equations

$$x + y = 6$$
$$6x + 3y = 30$$

For this system we can solve it by using the elimination method by multiplying the first equation by -3 and then adding it to the second equation.

$$-3(x + y = 6)$$
$$-3x - 3y = -18$$

Example 2 (Continued):

Step 4: Solve the system of equations

$$\begin{array}{r} -3x - 3y = -18 \\ \underline{6x + 3y = \quad 30} \\ 3x \qquad = \quad 12 \\ x = 4 \end{array}$$

Now that we know the value for x we can substitute it into the first equation to find y.

$$\begin{array}{r} x + y = 6 \\ 4 + y = 6 \\ y = 2 \end{array}$$

The blend would consist of 4 pounds of the \$6 grade and 2 pounds of the \$3 grade.