Review Exercise Set 23

Exercise 1: Determine the solution of the following system of equations.

$$2x - 3y + 5z = -5$$
$$2y - 3z = 4$$
$$4z = -8$$

Exercise 2: Determine the solution of the following system of equations.

$$2x + 3y - 4z = -10$$
$$2y + 3z = 16$$
$$2y - 5z = -16$$

Exercise 3: Determine the solution of the following system of equations.

$$x - 2y + 3z = 7$$
$$2x + y + 5z = 17$$
$$3x - 4y - 2z = 1$$

Exercise 4: Determine the solution of the following system of equations.

$$x - 2y + z = -4$$
$$2x + 4y - 3z = -1$$
$$-3x - 6y + 7z = 4$$

Exercise 5: Determine the solution of the following system of equations.

$$9x + 4y - z = 0$$
$$3x - 2y + 4z = 6$$
$$6x - 8y - 3z = 3$$

Review Exercise Set 23 Answer Key

Exercise 1: Determine the solution of the following system of equations.

$2x - 3y + 5z = -5$
$2y - 3z = 4$
$4z = -8$

Solve for z in the 3rd equation

$4z = -8$
$z = -2$

Substitute the value of z into the 2nd equation to find y

$2y - 3z = 4$
$2y - 3(-2) = 4$
$2y + 6 = 4$
$2y = 4 - 6$
$2y = -2$
$y = -1$

Substitute the values of y and z into the 1st equation to find x

$2x - 3y + 5z = -5$
$2x - 3(-1) + 5(-2) = -5$
$2x + 3 - 10 = -5$
$2x - 7 = -5$
$2x = -5 + 7$
$2x = 2$
$x = 1$

The solution for the system of equations is (1, -1, -2).

Exercise 2: Determine the solution of the following system of equations.

$2x + 3y - 4z = -10$
$2y + 3z = 16$
$2y - 5z = -16$

There is no equation with only one variable so we need to use two equations to try to eliminate one of the three variables. The 2nd and 3rd equations have the same two variables and the coefficients for y are the same in each equation, so we can multiply one of the equations by -1 and add it to the other to eliminate y.

$-1(2y - 5z = -16)$
$-2y + 5z = 16$

$$\begin{array}{r} 2y + 3z = 16 \\ \underline{-2y + 5z = 16} \\ 8z = 32 \\ z = 4 \end{array}$$

Substitute the value of z into the 2nd or 3rd equation to find y

$2y - 5z = -16$
$2y - 5(4) = -16$
$2y - 20 = -16$
$2y = -16 + 20$
$2y = 4$
$y = 2$

Substitute the values of y and z into the 1st equation to find x

$2x + 3y - 4z = -10$
$2x + 3(2) - 4(4) = -10$
$2x + 6 - 16 = -10$
$2x - 10 = -10$
$2x = -10 + 10$
$2x = 0$
$x = 0$

The solution for the system of equations is (0, 2, 4).

Exercise 3: Determine the solution of the following system of equations.

$x - 2y + 3z = 7$
$2x + y + 5z = 17$
$3x - 4y - 2z = 1$

The equations in this system all have three variables so we must add the equations together to derive two new equations with only two variables. Looking at the coefficients of the variables, y will be the easiest to eliminate.

Multiply the 2nd equation by 2 and add it to the 1st equation

$2(2x + y + 5z = 17)$
$4x + 2y + 10z = 34$

$x - 2y + 3z = 7$
$\underline{4x + 2y + 10z = 34}$
$5x + 13z = 41$ (equation #4)

Exercise 3 (Continued):

Multiply the 2nd equation by 4 and add it to the 3rd equation

$4(2x + y + 5z = 17)$
$8x + 4y + 20z = 68$

$3x - 4y - 2z = 1$
$\underline{8x + 4y + 20z = 68}$
$11x + 18z = 69$ (equation #5)

Now we must use the 4th and 5th equations to eliminate another of the variables. Since x has the smallest coefficients we will eliminate x.

Multiply the 4th equation by 11

$11(5x + 13z = 41)$
$55x + 143z = 451$

Multiply the 5th equation by -5

$-5(11x + 18z = 69)$
$-55x - 90z = -345$

Add these two equations together and solve for z

$55x + 143z = 451$
$\underline{-55x - 90z = -345}$
$53z = 106$
$z = 2$

Substitute the value of z into the 4th or 5th equation to find x

$5x + 13z = 41$
$5x + 13(2) = 41$
$5x + 26 = 41$
$5x = 41 - 26$
$5x = 15$
$x = 3$

Substitute the values of x and z into one of the original three equations to find y

$2x + y + 5z = 17$
$2(3) + y + 5(2) = 17$
$6 + y + 10 = 17$
$y + 16 = 17$
$y = 17 - 16$
$y = 1$

The solution for the system of equations is (3, 1, 2).

Exercise 4: Determine the solution of the following system of equations.

$$x - 2y + z = -4$$
$$2x + 4y - 3z = -1$$
$$-3x - 6y + 7z = 4$$

Multiply the 1st equation by -2 and add it to the 2nd equation

$$-2(x - 2y + z = -4)$$
$$-2x + 4y - 2z = 8$$

$$2x + 4y - 3z = -1$$
$$\underline{-2x + 4y - 2z = 8}$$
$$8y - 5z = 7 \text{ (equation \#4)}$$

Multiply the 1st equation by 3 and add it to the 3rd equation

$$3(x - 2y + z = -4)$$
$$3x - 6y + 3z = -12$$

$$-3x - 6y + 7z = 4$$
$$\underline{3x - 6y + 3z = -12}$$
$$-12y + 10z = -8 \text{ (equation \#5)}$$

Multiply the 4th equation by 2 and add it to the 5th equation

$$2(8y - 5z = 7)$$
$$16y - 10z = 14$$

$$-12y + 10z = -8$$
$$\underline{16y - 10z = 14}$$
$$4y = 6$$
$$y = \frac{3}{2}$$

Substitute the value of y into the 4th or 5th equation to find z

$$8y - 5z = 7$$
$$8\left(\frac{3}{2}\right) - 5z = 7$$
$$12 - 5z = 7$$
$$-5z = 7 - 12$$
$$-5z = -5$$
$$z = 1$$

Exercise 4 (Continued):

Substitute the values of y and z into one of the original three equations to find x

$x - 2y + z = -4$
$x - 2(\frac{3}{2}) + (1) = -4$
$x - 3 + 1 = -4$
$x - 2 = -4$
$x = -4 + 2$
$x = -2$

The solution for the system of equations is (-2, $\frac{3}{2}$, 1).

Exercise 5: Determine the solution of the following system of equations.

$9x + 4y - z = 0$
$3x - 2y + 4z = 6$
$6x - 8y - 3z = 3$

Multiply the 1st equation by 4 and add it to the 2nd equation

$4(9x + 4y - z = 0)$
$36x + 16y - 4z = 0$

$3x - 2y + 4z = 6$
$\underline{36x + 16y - 4z = 0}$
$39x + 14y = 6$ (equation #4)

Multiply the 1st equation by -3 and add it to the 3rd equation

$-3(9x + 4y - z = 0)$
$-27x - 12y + 3z = 0$

$6x - 8y - 3z = 3$
$\underline{-27x - 12y + 3z = 0}$
$-21x - 20y = 3$ (equation #5)

Multiply the 4th equation by 10

$10(39x + 14y = 6)$
$390x + 140y = 60$

Exercise 5 (Continued):

Multiply the 5th equation by 7

$7(-21x - 20y = 3)$
$-147x - 140y = 21$

Add these two equations together and solve for x

$390x + 140y = 60$
$\underline{-147x - 140y = 21}$
$243x = 81$
$x = \frac{81}{243}$
$x = \frac{1}{3}$

Substitute the value of x into the 4th or 5th equation to find y

$-21x - 20y = 3$
$-21(\frac{1}{3}) - 20y = 3$
$-7 - 20y = 3$
$-20y = 3 + 7$
$-20y = 10$
$y = \frac{10}{-20}$
$y = -\frac{1}{2}$

Substitute the values of x and y into one of the original three equations to find z

$9x + 4y - z = 0$
$9(\frac{1}{3}) + 4(-\frac{1}{2}) - z = 0$
$3 - 2 - z = 0$
$1 - z = 0$
$-z = -1$
$z = 1$

The solution for the system of equations is ($\frac{1}{3}$, -$\frac{1}{2}$, 1).