Review Exercise Set 24

Exercise 1: Jeremy is 10 years old than his baby sister, Amy. If the sum of their ages is 66, find their ages.

Exercise 2: Mercedes needs 6 pints of an acid that has a concentration of 40%. However, the only acid solutions available to her have concentrations of 20% and 50%. How much of each must she mix together to get the acid solution that she needs?
Exercise 3: Mila does not want to invest all of her lawsuit settlement ($40,000) into a single account, so she invests it into three different accounts. She is earning 7% interest on the amount she invested in the first account. She invested $5,000 less than what is in the first account into a second account earning 5% interest. The remaining balance is invested in the third account earning only 3% interest. If the total annual interest that Mila receives is $2,300, find the amount in each account.

Exercise 4: A pet store owner is looking to make a 40-pound mixture of birdseed that will cost $0.76 per pound by combining a generic wild bird seed that costs $0.59 per pound with blackoil sunflower seeds that cost $0.89 per pound. How many pounds of each seed must the owner blend together to get the desired mixture?
Exercise 1: Jeremy is 10 years older than his baby sister, Amy. If the sum of their ages is 66, find their ages.

Assign variables for each age

\[ x = \text{Jeremy's age} \]
\[ y = \text{Amy's age} \]

Translate the statements into equations for the system of equations

Jeremy is 10 years older than Amy
\[ x = 10 + y \]

The sum of their ages is 66
\[ x + y = 66 \]

Substitute the 1st equation into the 2nd for x and solve for y

\[ x + y = 66 \]
\[ (10 + y) + y = 66 \]
\[ 10 + 2y = 66 \]
\[ 2y = 66 - 10 \]
\[ 2y = 56 \]
\[ y = 28 \]

Substitute the value of y into the 1st equation to find x

\[ x = 10 + y \]
\[ x = 10 + 28 \]
\[ x = 38 \]

Jeremy is 38 years old and Amy is 28 years old.

Exercise 2: Mercedes needs 6 pints of an acid that has a concentration of 40%. However, the only acid solutions available to her have concentrations of 20% and 50%. How much of each must she mix together to get the acid solution that she needs?

Assign variables for each acid solution

\[ x = \text{amount of the 20% solution} \]
\[ y = \text{amount of the 50% solution} \]
Exercise 2 (Continued):

Use the table below to setup the equations

<table>
<thead>
<tr>
<th></th>
<th>Amount (A)</th>
<th>* Concentration (R)</th>
<th>= Quantity (Q)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20% Solution</td>
<td>x</td>
<td>* 20</td>
<td>= 20x</td>
</tr>
<tr>
<td>50% Solution</td>
<td>y</td>
<td>* 50</td>
<td>= 50y</td>
</tr>
<tr>
<td>40% Solution</td>
<td>6</td>
<td>* 40</td>
<td>= 240</td>
</tr>
</tbody>
</table>

\[ x + y = 6 \text{ (equation # 1)} \]
\[ 20x + 50y = 240 \]
\[ 2x + 5y = 24 \text{ (equation # 2)} \]

Multiply the 1st equation by -2

-2(x + y = 6)
-2x - 2y = -12

Add this new equation to the 2nd equation and solve for y

\[ 2x + 5y = 24 \]
\[ -2x - 2y = -12 \]
\[ 3y = 12 \]
\[ y = 4 \]

Substitute the value of y into the 1st equation to find x

\[ x + y = 6 \]
\[ x + 4 = 6 \]
\[ x = 6 - 4 \]
\[ x = 2 \]

Mercedes must mix together 2 pints of the 20% acid solution with 4 pints of the 50% acid solution.
Exercise 3: Mila does not want to invest all of her lawsuit settlement ($40,000) into a single account, so she invests it into three different accounts. She is earning 7% interest on the amount she invested in the first account. She invested $5,000 less than what is in the first account into a second account earning 5% interest. The remaining balance is invested in the third account earning only 3% interest. If the total annual interest that Mila receives is $2,300, find the amount in each account.

Assign variables for each account

\[ x = \text{amount invested in 1st account} \]
\[ y = \text{amount invested in 2nd account} \]
\[ z = \text{amount invested in 3rd account} \]

Use the table below to setup the equations

<table>
<thead>
<tr>
<th>Principal (P)</th>
<th>Rate (R)</th>
<th>Time (T)</th>
<th>Interest (I)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st account</td>
<td>x</td>
<td>.07</td>
<td>1</td>
</tr>
<tr>
<td>2nd account</td>
<td>y</td>
<td>.05</td>
<td>1</td>
</tr>
<tr>
<td>3rd account</td>
<td>z</td>
<td>.03</td>
<td>1</td>
</tr>
<tr>
<td>Total</td>
<td>40000</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The first equation will come from the principal column. The sum of the amounts deposited into each account must equal the $40,000 settlement.

\[ x + y + z = 40000 \text{ (equation #1)} \]

The next equation will come from the interest column. The total of the interest earned on each individual account must equal the total annual interest of $2,300. We can multiply the equation by 100 to get rid of the decimals.

\[ .07x + .05y + .03z = 2300 \]
\[ 100(.07x + .05y + .03z = 2300) \]
\[ 7x + 5y + 3z = 230000 \text{ (equation #2)} \]

The third equation will come from the statement that the amount invested in the second account was $5,000 less than what was invested in the first account.

\[ y = x - 5000 \text{ (equation #3)} \]

System of equations

\[ x + y + z = 40000 \]
\[ 7x + 5y + 3z = 230000 \]
\[ y = x - 5000 \]
Exercise 3 (Continued):

Substitute the 3rd equation into the first two equations to eliminate $y$ and reduce the equations to only two variables.

\[ x + y + z = 40000 \]
\[ x + (x - 5000) + z = 40000 \]
\[ 2x - 5000 + z = 40000 \]
\[ 2x + z = 40000 + 5000 \]
\[ 2x + z = 45000 \text{ (equation #4)} \]
\[ 7x + 5y + 3z = 230000 \]
\[ 7x + 5(x - 5000) + 3z = 230000 \]
\[ 7x + 5x - 25000 + 3z = 230000 \]
\[ 12x + 3z = 230000 + 25000 \]
\[ 12x + 3z = 255000 \text{ (equation #5)} \]

Multiply the 4th equation by -3 and add to the 5th equation to solve for $x$

\[-3(2x + z = 45000)\]
\[-6x - 3z = -135000\]
\[12x + 3z = 255000\]
\[-6x - 3z = -135000\]
\[6x = 120000\]
\[x = 20000\]

Substitute the value of $x$ into the 4th or 5th equation to find $z$

\[2x + z = 45000\]
\[2(20000) + z = 45000\]
\[40000 + z = 45000\]
\[z = 45000 - 40000\]
\[z = 5000\]

Substitute the value of $x$ into the 3rd equation to find $y$

\[y = x - 5000\]
\[y = 20000 - 5000\]
\[y = 15000\]

Mila invested $20,000 in the 1st account, $15,000 in the 2nd account, and $5,000 in the 3rd account.
Exercise 4: A pet store owner is looking to make a 40-pound mixture of birdseed that will cost $0.76 per pound by combining a generic wild bird seed that costs $0.59 per pound with black oil sunflower seeds that cost $0.89 per pound. How many pounds of each seed must the owner blend together to get the desired mixture?

Assign variables for each seed

\[ x = \text{amount of generic wild bird seed} \]
\[ y = \text{amount of black oil sunflower seed} \]

Use the table below to setup the equations

<table>
<thead>
<tr>
<th></th>
<th>Amount (A)</th>
<th>* Cost/pound (C)</th>
<th>= Value (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>generic seed</td>
<td>x</td>
<td>0.59</td>
<td>= 0.59x</td>
</tr>
<tr>
<td>sunflower</td>
<td>y</td>
<td>0.89</td>
<td>= 0.89y</td>
</tr>
<tr>
<td>mixture</td>
<td>40</td>
<td>0.76</td>
<td>= 30.4</td>
</tr>
</tbody>
</table>

The equations will come from the amount and value columns. The equation from the value column can be multiplied by 100 to get rid of the decimals.

\[ x + y = 40 \text{ (equation #1)} \]
\[ 0.59x + 0.89y = 30.4 \]
\[ 100(0.59x + 0.89y = 30.4) \]
\[ 59x + 89y = 3040 \text{ (equation #2)} \]

System of equations

\[ x + y = 40 \]
\[ 59x + 89y = 3040 \]

Multiply the 1st equation by -59 and add it to the 2nd equation to find y

\[ -59(x + y = 40) \]
\[ -59x - 59y = -2360 \]
\[ 59x + 89y = 3040 \]
\[ -59x - 59y = -2360 \]
\[ 30y = 680 \]
\[ y = \frac{680}{30} \]
\[ y = 22 \frac{2}{3} \]
Exercise 4 (Continued):

Substitute the value of $y$ into the 1st equation to find $x$

\[ x + y = 40 \]
\[ x + 22\frac{2}{3} = 40 \]
\[ x = 40 - 22\frac{2}{3} \]
\[ x = 17\frac{1}{3} \]

The pet store owner needs to use $17\frac{1}{3}$ pounds of the generic wild bird seed and $22\frac{2}{3}$ pounds of the black oil sunflower seed to make the desired mixture.