Solving Systems of Equations by the Addition Method

The elimination method may be used to solve systems of linear equations of more than two variables. The objective is to find the solution of the ordered triple (*x*, *y*, *z*) by using the elimination method covered earlier. The following example demonstrates this idea.

Example 1: Solve the system of equations.

$$x - y + 4z = -29 \quad (1)$$
$$3x - 3y - z = -6 \quad (2)$$
$$2x - 5y + 6z = -55 \quad (3)$$

Solution:

Step 1: Create a 4th equation by eliminating any variable between any two of the given equations. In this case the "x" variable will be eliminated in the combination of equations 1 and 2. This is done by multiplying equation 1 by -3 and then adding it to equation 2.

$$\begin{array}{r} 3x - 2y - z = -6 \\ -3x + 3y - 12z = 87 \\ \hline y - 13z = 81 \end{array} \quad (4)$$

Step 2: Create a 5th equation by taking the remaining unused original equation and any one of the other two equations and eliminate the same variable as in step 1. In this case equations 1 and 3 are used with the goal of eliminating the "x" variable. Equation 5 will be the sum of equation 3 and -2 times equation 1.

$$\begin{array}{r} 2x - 5y + 6z = -55 \\ -2x + 2y - 8z = 58 \\ \hline -3y - 2z = 3 \end{array} \quad (5)$$

Step 3: Using equations 4 and 5 the process is repeated to eliminate either of the two remaining variables. For this example "y" will be eliminated by first multiplying equation 4 by 3 and then added to equation 5.

$$\begin{array}{r} -3y - 2z = 3 \\ 3y - 39z = 243 \\ \hline -41z = 246 \end{array}$$

Example 1 (Continued):

Step 4: Solve for z.

$$-41z = 246$$
$$\frac{-41z}{-41} = \frac{246}{-41}$$
$$z = -6$$

Step 5: Solve for y.

The solution of step 4 is substituted into either equations 4 or 5 to solve for the second variable. In this case equation 5 will be used.

$$-3y - 2z = 3$$
$$-3y - 2(-6) = 3$$
$$-3y - (-12) = 3$$
$$-3y + 12 = 3$$
$$-3y + 12 - 12 = 3 - 12$$
$$-3y = -9$$
$$y = 3$$

Step 6: Solve for x.

The solutions found in steps 4 and 5 are substituted into any of the original 3 equations to solve for the remaining variable. In this case equation 1 will be used.

$$x - y + 4z = -29$$
$$x - (3) + 4(-6) = -29$$
$$x - 3 - 24 = -29$$
$$x - 27 = -29$$
$$x = -2$$

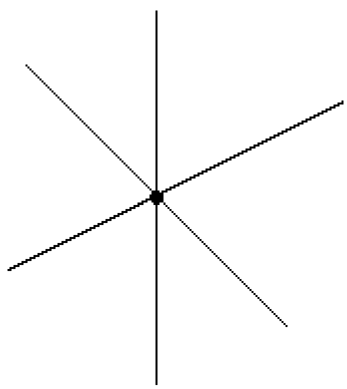
Example 1 (Continued):

Step 7: Analysis.

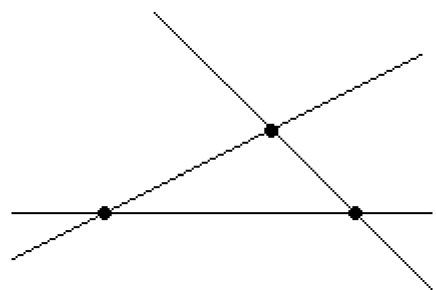
The solutions found for the ordered triple are substituted into the two remaining equations to verify their validity.

$$\begin{aligned} 3x - 2y - z &= -6 \\ 3(-2) - 2(3) - (-6) &= -6 \\ -6 - 6 + 6 &= -6 \\ -6 &= -6 \end{aligned} \qquad \begin{aligned} 2x - 5y + 6z &= -55 \\ 2(-2) - 5(3) + 6(-6) &= -55 \\ -4 - 15 - 36 &= -55 \\ -55 &= -55 \end{aligned}$$

The verification is necessary since there are a number of ways in which the system may behave.

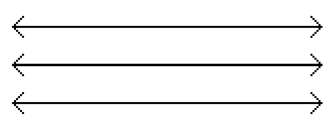


One solution



Multiple solutions

Infinite solutions



No solution