Solving Systems of Equations by Substitution

Throughout this tutorial, we have dealt with problems that have one equation and usually one variable to work with. However, many times in algebra we have to deal with problems which give rise to sets of equations with several variables. If the particular problem gives us a set of equations, which have the same variables, then we call the set a *system of equations.* Now, we must develop techniques for finding the solutions set for multiple variables that will work for all equations within the system of equations.

We can solve a system of equations by substitution, by elimination, or graphically. We will look at the substitution method in this section

Substitution Method

In the substitution method, we start with one equation in the system and solve for one variable in terms of the other variable. We then substitute the result into the other equation. The following box describes the procedure more explicitly.

SUBSTITUTION METHOD

1. SOLVE FOR ONE VARIABLE. Choose one equation and solve for one variable in terms of the other variable.

2. SUBSTITUTE. Substitute the expression you found in Step 1 into the other equation to get an equation in one variable, and then solve for that variable.

3. BACK-SUBSTITUTE. Substitute the value you found in Step 2 back into the expression found in Step 1 to solve for the remaining variable.

Example 1: Find all solutions of the system.

$$\begin{cases} 2x + y = 7 \\ 3x - y = 13 \end{cases}$$

Solution:

Step 1: We solve for *y* in the first equation.

$$y = 7 - 2x$$

Example 1 (Continued):

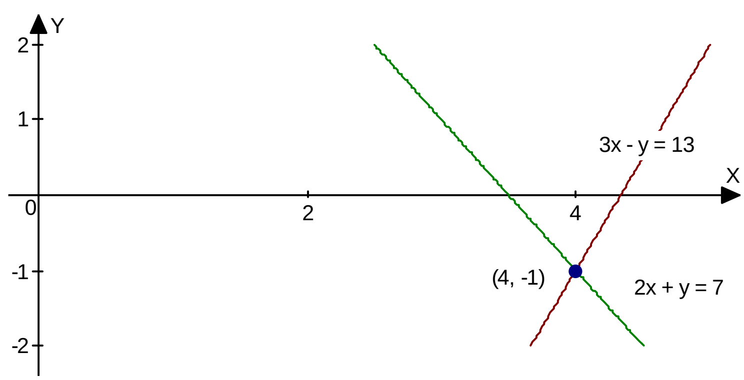
Step 2: **Now we substitute for *y* in the second equation and then solve for *x*:**

$$3x-(7-2x)=13 \quad \text{Substitute } y=7-2x \text{ into the second equation}$$
$$3x-7+2x=13 \quad \text{Expand}$$
$$5x-7=13 \quad \text{Simplify}$$
$$5x=20 \quad \text{Add 7}$$
$$x=4 \quad \text{Solve for } x$$

Step 3: **Next we back-substitute *x* = 4 into the equation**

$$y = 7 - 2x$$
$$y = 7 - 2(4)$$
$$y = -1$$

Thus, $x = 4$ and $y = -1$, so the solution is the ordered pair (4, -1). The following figure shows that the graphs of the two equations intersect at the point (4, -1).



Step 4: **Lastly, we check our answer.**

$$x=4, y=-1:$$
$$\begin{cases} 2(4)+(-1)=7 \\ 3(4)-(-1)=13 \end{cases}$$

Example 2: Find all solutions of the system.

$$\begin{cases} x^2 + y^2 = 100 \\ 3x - y = 10 \end{cases}$$

Solution:

Step 1: **We start by solving for *y* in the second equation.**

$y = 3x - 10$ Solve for y in the second equation

Step 2: **Next, we substitute for *y* in the first equation and solve for *x*:**

$x^2 + (3x - 10)^2 = 8$ Substitute $y = 3x - 10$ into the 1st equation

$x^2 + (9x^2 - 60x + 100) = 8$ Expand

$10x^2 - 60x = 0$ Simplify

$10x(x - 6) = 0$ Factor

$x = 0$ or $x = 6$ Solve for x

Step 3: **Now we back-substitute these values of *x* into the equation *y* = 3*x* – 10.**

For $x = 0$: $y = 3(0) - 10 = -10$ Back substitute

For $x = 6$: $y = 3(6) - 10 = 8$ Back substitute

So, we have two solutions: (0, -10) and (6, 8). We still need to check our answer to make sure these solutions have been found properly.

Checking your answers:

$x = 0, y = -10$:

$$\begin{cases} 0^2 + (-10)^2 = 100 \\ 3(0) - (-10) = 10 \end{cases}$$

$x = 6, y = 8$:

$$\begin{cases} 6^2 + 8^2 = 100 \\ 3(6) - 8 = 10 \end{cases}$$

Everything checks out to be correct.

Example 2 (Continued):

The graph of the first equation is a circle, and the graph of the second equation is a line; the following figure shows that the graphs intersect at two points (0, -10) and (6, 8).