Review Exercise Set 10

Exercise 1:  Solve the exponential expression exactly for x.

\[ 4^x = 256 \]

Exercise 2:  Use a calculator to solve the exponential expression for x. Round your answer to three decimal places.

\[ e^{2x} + 2e^x - 15 = 0 \]

Exercise 3:  Find the exact value of x in the logarithmic equation. Do not round off the answer.

\[ \log (x - 1) + \log (x + 6) = \log (4x) \]
Exercise 4: Use a calculator to solve the exponential expression for \(x\). Round your answer to three decimal places.

\[
\ln (1 - x) - \ln (2x + 3) = \ln (x)
\]

Exercise 5: If you were to deposit $5,000 into an account paying 6% interest compounded quarterly, how many years would it take until you had $30,000 in the account?
Review Exercise Set 10 Answer Key

Exercise 1: Solve the exponential expression exactly for x.

\[ 4^x = 256 \]
\[ 4^x = 4^4 \]
\[ x = 4 \]

Exercise 2: Use a calculator to solve the exponential expression for x. Round your answer to three decimal places.

\[ e^{2x} + 2e^x - 15 = 0 \]

To make the problem look easier, let \( w = e^x \) which would make \( e^{2x} = w^2 \)

\[ w^2 + 2w - 15 = 0 \]

Factor the quadratic equation

\[ (w + 5)(w - 3) = 0 \]

Set each factor equal to zero

\[ w + 5 = 0 \text{ or } w - 3 = 0 \]

Replace \( w \) with \( e^x \) and solve each equation for x

\[ e^x + 5 = 0 \text{ or } e^x - 3 = 0 \]

\[ e^x = -5 \]
\[ \ln e^x = \ln (-5) \]
\[ x = \ln (-5) \]

\( \ln (-5) \) is not a real number so there is no solution for this equation

\[ e^x - 3 = 0 \]
\[ e^x = 3 \]
\[ \ln e^x = \ln 3 \]
\[ x = \ln 3 \]
\[ x \approx 1.099 \]
Exercise 3: Find the exact value of x in the logarithmic equation. Do not round off the answer.

\[
\log(x - 1) + \log(x + 6) = \log(4x)
\]

Combine the logs on the left side of the equation

\[
\log((x - 1)(x + 6)) = \log(4x)
\]

Use the properties of logs to remove the logs from both sides

\[
(x - 1)(x + 6) = 4x
\]

FOIL the left side

\[
x^2 + 5x - 6 = 4x
\]

Gather all terms on the left side

\[
x^2 + x - 6 = 0
\]

Factor and solve for x

\[
(x + 3)(x - 2) = 0
\]

\[
x + 3 = 0
\]

\[
x = -3
\]

\[
x - 2 = 0
\]

\[
x = 2
\]

Test each value to determine if it is a valid solution

\[
\log(x - 1) + \log(x + 6) = \log(4x)
\]

\[
\log(-3 - 1) + \log(-3 + 6) = \log(4)(-3)
\]

\[
\log(-4) + \log(3) = \log(-12)
\]

\[
x = -3\) is not a valid solution since it would cause us to be taking the log of a negative number.

\[
\log(x - 1) + \log(x + 6) = \log(4x)
\]

\[
\log(2 - 1) + \log(2 + 6) = \log(4)(2)
\]

\[
\log(1) + \log(8) = \log(8)
\]

\[
\log(1 * 8) = \log(8)
\]

\[
\log(8) = \log(8)
\]

\[
x = 2\) would be the solution for this logarithmic equation.
Exercise 4:  Use a calculator to solve the exponential expression for $x$. Round your answer to three decimal places.

\[
\ln (1 - x) - \ln (2x + 3) = \ln (x)
\]
\[
\ln (1 - x) = \ln (x) + \ln (2x + 3)
\]
\[
\ln (1 - x) = \ln (x)(2x + 3)
\]
\[
1 - x = (x)(2x + 3)
\]
\[
1 - x = 2x^2 + 3x
\]
\[
0 = 2x^2 + 4x - 1
\]

The quadratic equation will not factor so we must use the quadratic formula.

\[
a = 2; \ b = 4; \ c = -1
\]
\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]
\[
= \frac{-4 \pm \sqrt{(4)^2 - 4(2)(-1)}}{2(2)}
\]
\[
= \frac{-4 \pm \sqrt{16 + 8}}{4}
\]
\[
= \frac{-4 \pm \sqrt{24}}{4}
\]
\[
= \frac{-4 \pm 2\sqrt{6}}{4}
\]
\[
= \frac{-2 \pm \sqrt{6}}{2}
\]

\[x = \frac{-2 + \sqrt{6}}{2} \text{ or } x = \frac{-2 - \sqrt{6}}{2}\]
\[x \approx 0.225 \quad \text{or} \quad x \approx -2.225\]

The variable $x$ cannot be a negative number since it would cause us to have the natural log of a negative number which is not possible. Therefore, the solution for the equation is that $x$ is approximately equal to 0.225.
Exercise 5: If you were to deposit $5,000 into an account paying 6% interest compounded quarterly, how many years would it take until you had $30,000 in the account?

Identify the given information:

- \( P = 5000 \)
- \( r = 6\% = 0.06 \)
- \( n = 4 \) (quarterly)
- \( A = 30000 \)

Substitute the known values into the compound interest formula and solve for the unknown variable.

\[
A = P \left(1 + \frac{r}{n}\right)^{nt}
\]

\[
30000 = 5000 \left(1 + \frac{0.06}{4}\right)^{4t}
\]

\[
30000 = 5000(1.015)^{4t}
\]

\[
6 = (1.015)^{4t}
\]

\[
\log 6 = \log (1.015)^{4t}
\]

\[
\log 6 = 4t \times \log 1.015
\]

\[
\frac{\log 6}{4 \log 1.015} = t
\]

\[
30.086 \approx t
\]

It would take approximately 30 years for the account balance to reach $30,000.