Review Exercise Set 7

Exercise 1: Approximate the value of each exponential expression.

a) $e^3$  

b) $e^0$  

c) $e^{1/4}$

Exercise 2: Find the $y$-intercept and horizontal asymptote for the following exponential function.

$$f(x) = e^{-0.1x} + 2$$

Exercise 3: Find the $y$-intercept and horizontal asymptote for the following exponential function.

$$f(x) = e^{x/2}$$
Exercise 4: Sketch the graph of the following exponential function by locating the asymptote, y-intercept, and other points on the graph.

\[ f(x) = e^{-x} - 3 \]

Exercise 5: A small town with 10,000 residents has been experiencing an increase in their population at a rate of 3% a year. If this growth in population continues how many residents will the town have in 10 years? (Hint: \( N = N_0e^{rt} \))
Exercise 1: Approximate the value of each exponential expression.

a) $e^3$  
\[ e^3 = 2.7183^3 \approx 20.09 \]

b) $e^0$  
\[ e^0 = 1 \]

c) $e^{1/4}$  
\[ e^{1/4} = \sqrt[4]{e} \approx 1.28 \]

Exercise 2: Find the y-intercept and horizontal asymptote for the following exponential function.

\[ f(x) = e^{-0.1x} + 2 + 2 \]

**Y-intercept**

let $x = 0$

\[ f(x) = e^{-0.1(0)} + 2 + 2 \]
\[ f(0) = e^0 + 2 \]
\[ f(0) = e^2 + 2 \]
\[ f(0) \approx 7.39 + 2 \]
\[ f(0) \approx 9.39 \]

**The y-intercept is approximately located at (0, 9.4)**

**Horizontal asymptote**

For the base function $e^x$ the horizontal asymptote is the x-axis ($y = 0$). The given function for $f(x)$ contains a vertical shift of +2. Therefore, the horizontal asymptote will be shifted up 2 units to $y = 2$.

**The horizontal asymptote is at $y = 2$**
Exercise 3: Find the y-intercept and horizontal asymptote for the following exponential function.

\[ f(x) = e^{x/2} \]

Y-intercept

Let \( x = 0 \)

\[ f(x) = e^{x/2} \]
\[ f(0) = e^{0/2} \]
\[ f(0) = e^0 \]
\[ f(0) = 1 \]

The y-intercept is located at \((0, 1)\)

Horizontal asymptote

For the base function \( e^x \) the horizontal asymptote is the x-axis (\( y = 0 \)). The given function for \( f(x) \) does not contain a vertical shift so the horizontal asymptote will remain at the x-axis.

The horizontal asymptote is at \( y = 0 \)

Exercise 4: Sketch the graph of the following exponential function by locating the asymptote, y-intercept, and other points on the graph.

\[ f(x) = e^{-x} - 3 \]

Y-intercept

Let \( x = 0 \)

\[ f(x) = e^{-x} - 3 \]
\[ f(0) = e^{-(0)} - 3 \]
\[ f(0) = e^0 - 3 \]
\[ f(0) = 1 - 3 \]
\[ f(0) = -2 \]

The y-intercept is located at \((0, -2)\)

Horizontal asymptote

For the base function \( e^x \) the horizontal asymptote is the x-axis (\( y = 0 \)). The given function for \( f(x) \) has a vertical shift of -3 so the horizontal asymptote will be shifted downward 3 from the x-axis.

The horizontal asymptote is at \( y = -3 \)
Exercise 4 (Continued):

Additional points

<table>
<thead>
<tr>
<th>x</th>
<th>f(x)</th>
<th>f(x)</th>
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<tbody>
<tr>
<td>-2</td>
<td>e^{-2} - 3</td>
<td>e^{-2} - 3</td>
</tr>
<tr>
<td></td>
<td>≈ 4.389</td>
<td>≈ 0.135</td>
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Graph
Exercise 5: A small town with 10,000 residents has been experiencing an increase in their population at a rate of 3% a year. If this growth in population continues how many residents will the town have in 10 years? (Hint: $N = N_0e^{rt}$)

Identify given information

\[ N_0 = 10000 \]
\[ r = 3\% = 0.03 \]
\[ t = 10 \]

Substitute the given data into the formula to solve for the unknown value.

\[ N = N_0e^{rt} \]
\[ N = (10000)e^{0.03\times10} \]
\[ N = 10000e^{0.3} \]
\[ N \approx 10000(1.34985) \]
\[ N \approx 13498.5 \]

In 10 years the population will be approximately 13,499 residents.