

Review Exercise Set 12

Exercise 1: Using the given augmented matrix, perform the indicated row operation.

$$\left[\begin{array}{cc|c} 1 & 2 & 5 \\ -2 & 3 & 4 \end{array} \right]$$

$$2R_1 + R_2 \rightarrow R_2$$

Exercise 2: Using the given augmented matrix, perform the indicated row operation.

$$\left[\begin{array}{ccc|c} 1 & -2 & -1 & 2 \\ 0 & 2 & 2 & -1 \\ 2 & 4 & -2 & 4 \end{array} \right]$$

$$-2R_1 + R_3 \rightarrow R_3$$

Exercise 3: Use row operations to transform the given augmented matrix to reduced form.

$$\left[\begin{array}{cc|c} -1 & 3 & 7 \\ 2 & 5 & -12 \end{array} \right]$$

Exercise 4: Solve the system of linear equations using an augmented matrix and row operations.

$$2x + 3y + 2z = 1$$

$$-y + z = -2$$

$$x + y - z = 6$$

Review Exercise Set 12 Answer Key

Exercise 1: Using the given augmented matrix, perform the indicated row operation.

$$\left[\begin{array}{cc|c} 1 & 2 & 5 \\ -2 & 3 & 4 \end{array} \right]$$

$$2R_1 + R_2 \rightarrow R_2$$

First, multiply the elements in row one by 2

$$2[1 \ 2 \ | \ 5] = [2 \ 4 \ | \ 10]$$

Now, add the elements from row two

$$\begin{aligned} [2 \ 4 \ | \ 10] + [-2 \ 3 \ | \ 4] &= [(2+(-2)) \ (4+3) \ | \ (10+4)] \\ &= [0 \ 7 \ | \ 14] \end{aligned}$$

Replace the existing row two in our matrix with the new row two

$$\left[\begin{array}{cc|c} 1 & 2 & 5 \\ 0 & 7 & 14 \end{array} \right]$$

Exercise 2: Using the given augmented matrix, perform the indicated row operation.

$$\left[\begin{array}{ccc|c} 1 & -2 & -1 & 2 \\ 0 & 2 & 2 & -1 \\ 2 & 4 & -2 & 4 \end{array} \right]$$

$$-2R_1 + R_3 \rightarrow R_3$$

Multiply row one by -2

$$\begin{aligned} -2R_1 &= -2[1 \ -2 \ -1 \ | \ 2] \\ &= [-2 \ 4 \ 2 \ | \ -4] \end{aligned}$$

Add row three

$$\begin{aligned} [-2 \ 4 \ 2 \mid -4] + [2 \ 4 \ -2 \mid 4] &= [(-2+2) \ (4+4) \ (2+(-2)) \mid (-4+4)] \\ &= [0 \ 8 \ 0 \mid 0] \end{aligned}$$

Replacing existing row three with the new row three

$$\left[\begin{array}{ccc|c} 1 & -2 & -1 & 2 \\ 0 & 2 & 2 & -1 \\ 0 & 8 & 0 & 0 \end{array} \right]$$

Exercise 3: Use row operations to transform the given augmented matrix to reduced form.

$$\left[\begin{array}{cc|c} -1 & 3 & 7 \\ 2 & 5 & -12 \end{array} \right]$$

Multiply row one by -1 and use the result as the new row one.

$$-1R_1 \rightarrow R_1$$

$$-1[-1 \ 3 \mid 7] = [1 \ -3 \mid -7]$$

$$\left[\begin{array}{cc|c} 1 & -3 & -7 \\ 2 & 5 & -12 \end{array} \right]$$

Multiply row one by -2 and add to row two to obtain the new row two.

$$-2R_1 + R_2 \rightarrow R_2$$

$$\begin{aligned} -2[1 \ -3 \mid -7] + [2 \ 5 \mid -12] &= [2 \ -6 \mid -14] + [2 \ 5 \mid -12] \\ &= [0 \ 11 \mid 2] \end{aligned}$$

$$\left[\begin{array}{cc|c} 1 & -3 & -7 \\ 0 & 11 & 2 \end{array} \right]$$

Exercise 3 (Continued):

Divide row two by 11 and use the result as the new row two.

$$R_2 \div 11 \rightarrow R_2$$

$$\begin{aligned} \left[0 \quad 11 \mid 2 \right] \div 11 &= \left[\frac{0}{11} \quad \frac{11}{11} \mid \frac{2}{11} \right] \\ &= \left[0 \quad 1 \mid \frac{2}{11} \right] \end{aligned}$$

$$\left[\begin{array}{cc|c} 1 & -3 & -7 \\ 0 & 1 & \frac{2}{11} \end{array} \right]$$

Multiply row two by 3 and add to row one to obtain the new row one.

$$3R_2 + R_1 \rightarrow R_1$$

$$\begin{aligned} 3 \left[0 \quad 1 \mid \frac{2}{11} \right] + [1 \quad -3 \mid -7] &= \left[0 \quad 3 \mid \frac{6}{11} \right] + [1 \quad -3 \mid -7] \\ &= \left[(0+1) \quad (3+(-3)) \mid \left(\frac{6}{11} + (-7) \right) \right] \\ &= \left[1 \quad 0 \mid -6\frac{5}{11} \right] \end{aligned}$$

$$\left[\begin{array}{cc|c} 1 & 0 & -6\frac{5}{11} \\ 0 & 1 & \frac{2}{11} \end{array} \right]$$

The solution set for the system of equations is $(-6\frac{5}{11}, 2/11)$

Exercise 4: Solve the system of linear equations using an augmented matrix and row operations.

$$\begin{aligned} 2x + 3y + 2z &= 1 \\ -y + z &= -2 \\ x + y - z &= 6 \end{aligned}$$

Add any missing terms using a zero coefficient

$$\begin{aligned} 2x + 3y + 2z &= 1 \\ 0x - y + z &= -2 \\ x + y - z &= 6 \end{aligned}$$

Exercise 4 (Continued):

Setup the augmented matrix

$$\left[\begin{array}{ccc|c} 2 & 3 & 2 & 1 \\ 0 & -1 & 1 & -2 \\ 1 & 1 & -1 & 6 \end{array} \right]$$

Switch rows one and three

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 6 \\ 0 & -1 & 1 & -2 \\ 2 & 3 & 2 & 1 \end{array} \right]$$

Multiply row one by -2 and add to row three to obtain new row three

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 6 \\ 0 & -1 & 1 & -2 \\ 0 & 1 & 4 & -11 \end{array} \right]$$

Add rows one and two to obtain new row one

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 4 \\ 0 & -1 & 1 & -2 \\ 0 & 1 & 4 & -11 \end{array} \right]$$

Add rows two and three to obtain new row three

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 4 \\ 0 & -1 & 1 & -2 \\ 0 & 0 & 5 & -13 \end{array} \right]$$

Multiply row two by -1 to obtain new row two

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 4 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & 5 & -13 \end{array} \right]$$

Exercise 4 (Continued):

Divide row three by 5 to obtain new row three

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 4 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & 1 & -2\frac{3}{5} \end{array} \right]$$

Add rows two and three to obtain new row two

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -\frac{3}{5} \\ 0 & 0 & 1 & -2\frac{3}{5} \end{array} \right]$$

Solution set for the system of equations is $(x, y, z) = (4, -3/5, -2\ 3/5)$