Review Exercise Set 4

Exercise 1: Use the rational zero theorem to list the possible rational zeros for the given polynomial.

$$f(x) = x^6 - 2x^5 + 3x^2 + 9x - 42$$

Exercise 2: Use the factor theorem to determine if the given number is a zero of the polynomial.

$$f(x) = x^3 - 4x^2 + 2x - 8; x = 2$$

Exercise 3: Use Descartes's rule of signs to determine the possible number of positive and negative real zeros for the given polynomial.

$$f(x) = 4x^4 - 16x^3 + 12x^2 + x - 9$$

Exercise 4: Use synthetic division to find the value of f(1) for the polynomial

$$f(x) = 6x^5 + x^4 - 7x^2 + x - 1.$$

Exercise 5: Given the following zero of the polynomial find all of the others. Write the polynomial as a product of linear factors.

$g(x) = 2x^3 + 3x^2 - 3x - 2$; 1 is a zero

Review Exercise Set 4 Answer Key

Exercise 1: Use the rational zero theorem to list the possible rational zeros for the given polynomial.

$f(x) = x^6 - 2x^5 + 3x^2 + 9x - 42$

factors of p (-42): ±1, ±2, ±3, ±6, ±7, ±14, ±21, ±42
factors of q (1): ±1

possible rational zeros (p/q): ±1, ±2, ±3, ±6, ±7, ±14, ±21, ±42

Exercise 2: Use the factor theorem to determine if the given number is a zero of the polynomial.

$f(x) = x^3 - 4x^2 + 2x - 8; x = 2$

$f(x) = x^3 - 4x^2 + 2x - 8$
$f(2) = (2)^3 - 4(2)^2 + 2(2) - 8$
$f(2) = 8 - 4(4) + 4 - 8$
$f(2) = 8 - 16 + 4 - 8$
$f(2) = -12$

$f(2) \neq 0$ so x = 2 is not a zero of the polynomial.

Exercise 3: Use Descartes's rule of signs to determine the possible number of positive and negative real zeros for the given polynomial.

$f(x) = 4x^4 - 16x^3 + 12x^2 + x - 9$

$f(x) = 4x^4 - 16x^3 + 12x^2 + x - 9$
3 sign changes so there are either 3 or 1 positive real zeros

$f(-x) = 4(-x)^4 - 16(-x)^3 + 12(-x)^2 + (-x) - 9$
$f(-x) = 4x^4 + 16x^3 + 12x^2 - x - 9$
$f(-x) = 4x^4 + 16x^3 + 12x^2 - x - 9$
Only 1 sign change so there is 1 negative real zero

Exercise 4: Use synthetic division to find the value of f(1) for the polynomial

$f(x) = 6x^5 + x^4 - 7x^2 + x - 1.$

First, rewrite the polynomial to include any missing terms. Any missing terms added to the polynomial must have a coefficient of zero.

$f(x) = 6x^5 + x^4 - 7x^2 + x - 1$
$f(x) = 6x^5 + x^4 + 0x^3 - 7x^2 + x - 1$

Perform synthetic division letting x = 1

$$\begin{array}{r|rrrrrr} 1 & 6 & 1 & 0 & -7 & 1 & -1 \\ & & 6 & 7 & 7 & 0 & 1 \\ \hline & 6 & 7 & 7 & 0 & 1 & \color{red}{0} \end{array}$$

f(1) = 0

Exercise 5: Given the following zero of the polynomial find all of the others. Write the polynomial as a product of linear factors.

$g(x) = 2x^3 + 3x^2 - 3x - 2$; 1 is a zero

Perform synthetic division letting x = 1

$$\begin{array}{r|rrrr} 1 & 2 & 3 & -3 & -2 \\ & & 2 & 5 & 2 \\ \hline & 2 & 5 & 2 & \color{red}{0} \end{array}$$

Now, write the polynomial g(x) with the first factor taken out. We performed the synthetic division letting x = 1 so our factor is (x - 1)

$x = 1$
$x - 1 = 0$

The remaining polynomial will come from the three coefficients to the left of the remainder of zero. Remember to reduce the power of the leading term by one from the original polynomial since we have taken out one factor so far.

$$\begin{array}{r|rrrr} 1 & 2 & 3 & -3 & -2 \\ & & 2 & 5 & 2 \\ \hline & 2 & 5 & 2 & \color{red}{0} \\ & \downarrow & \downarrow & \downarrow & \end{array}$$

$$2x^2 + 5x + 2$$

$g(x) = (x - 1)(2x^2 + 5x + 2)$

Exercise 5 (Continued):

Factor the quadratic factor in our polynomial.

$g(x) = (x - 1)(2x^2 + 5x + 2)$

$\mathbf{g(x) = (x - 1)(2x + 1)(x + 2)}$