Review Exercise Set 5

Exercise 1: Find the domain for the given rational function.

$$f(x) = \frac{3x^2 + x}{x^2 + 4}$$

Exercise 2: Find the domain for the given rational function.

$$f(x) = \frac{3x^2}{2x^2 - 5x - 3}$$

Exercise 3: Find all vertical and horizontal asymptotes (if any) for the given rational function.

$$f(x) = \frac{3x + 4}{x - 5}$$

Exercise 4: Find all vertical and horizontal asymptotes (if any) for the given rational function.

$$f(x) = \frac{x^2 + 4x + 4}{x^3 - x^2 - 6x}$$

Exercise 5: Determine the x and y intercepts, vertical and horizontal asymptotes, symmetry, and any additional points needed to graph the given rational function.

$$f(x) = \frac{2x^2 - 18}{-3x^2 - 12x - 12}$$

Review Exercise Set 5 Answer Key

Exercise 1: Find the domain for the given rational function.

$$f(x) = \frac{3x^2 + x}{x^2 + 4}$$

To determine if we have any domain restrictions, set the denominator equal to zero and solve for x.

$x^2 + 4 = 0$
$x^2 = -4$

x^2 will never be equal to a negative number so there are no domain restrictions. The domain for this rational function is all real numbers.

D = $(-\infty, \infty)$ -- interval notation
or D = $\{x \mid x \in \mathbb{R}\}$ -- set notation

Exercise 2: Find the domain for the given rational function.

$$f(x) = \frac{3x^2}{2x^2 - 5x - 3}$$

Set denominator equal to zero and solve for x

$2x^2 - 5x - 3 = 0$
$(2x + 1)(x - 3) = 0$

Set each factor equal to zero

$2x + 1 = 0$ or $x - 3 = 0$
$x = -\frac{1}{2}$ or $x = 3$

Our domain restrictions are that x cannot be equal to -1/2 or 3, so our domain will be all real numbers except for these two.

D = $\left(-\infty, -\frac{1}{2}\right) \cup \left(-\frac{1}{2}, 3\right) \cup (3, \infty)$
or D = $\left\{x \mid x \neq -\frac{1}{2} \text{ or } 3\right\}$

Exercise 3: Find all vertical and horizontal asymptotes (if any) for the given rational function.

$$f(x) = \frac{3x + 4}{x - 5}$$

To find the vertical asymptote, set the denominator equal to zero and solve for x.

$x - 5 = 0$
$x = 5$

To find the horizontal asymptote, compare the degree power of the numerator and denominator.

numerator is $3x^1 + 4$ with a degree of 1
denominator is $x^1 - 5$ with a degree of 1

Since the degree powers are the same the horizontal asymptote will be equal to the ratio of the leading coefficients in the numerator and denominator.

$$f(x) = \frac{3x + 4}{1x - 5}$$

$y = \frac{3}{1}$
$y = 3$

The asymptotes are:

vertical: x = 5
horizontal: y = 3

Exercise 4: Find all vertical and horizontal asymptotes (if any) for the given rational function.

$$f(x) = \frac{x^2 + 4x + 4}{x^3 - x^2 - 6x}$$

Vertical asymptotes

$x^3 - x^2 - 6x = 0$
$x(x^2 - x - 6) = 0$
$x(x - 3)(x + 2) = 0$

$x = 0$ or $x - 3 = 0$ or $x + 2 = 0$
$x = 0$ or $x = 3$ or $x = -2$

Exercise 4 (Continued):

Horizontal asymptotes

numerator is $x^2 + 4x + 4$ with a degree of 2
denominator is $x^3 - x^2 - 6x$ with a degree of 3

Since the degree power of the denominator is one more than the numerator, the horizontal asymptote will be the x-axis.

$y = 0$

The asymptotes are:

vertical: x = -2, x = 0, and x = 3
horizontal: y = 0

Exercise 5: Determine the x and y intercepts, vertical and horizontal asymptotes, symmetry, and any additional points needed to graph the given rational function.

$$f(x) = \frac{2x^2 - 18}{-3x^2 - 12x - 12}$$

Set f(x) = 0 and solve for x to find the x-intercepts

$$0 = \frac{2x^2 - 18}{-3x^2 - 12x - 12}$$

$0 = 2x^2 - 18$
$18 = 2x^2$
$9 = x^2$
$\pm 3 = x$

x-intercepts are (-3,0) and (3, 0)

Substitute 0 for x to find the y-intercept

$$f(x) = \frac{2x^2 - 18}{-3x^2 - 12x - 12}$$

$$f(0) = \frac{2(0)^2 - 18}{-3(0)^2 - 12(0) - 12}$$

$$= \frac{-18}{-12}$$

$$= \frac{3}{2}$$

y-intercept is (0, 3/2)

Exercise 5 (Continued):

Set denominator equal to zero and solve for x to find vertical asymptotes

$-3x^2 - 12x - 12 = 0$
$-3(x^2 + 4x + 4) = 0$
$-3(x + 2)(x + 2) = 0$
$x + 2 = 0$
$x = -2$

vertical asymptote is at x = -2

Compare degree powers to determine horizontal asymptote

numerator is $2x^2 - 18$ with a degree of 2
denominator is $-3x^2 - 12x - 12$ with a degree of 2

Since the degree powers are the same, the horizontal asymptote will be the ratio of the leading coefficients (2 and -3).

$y = \frac{2}{-3}$

$y = -\frac{2}{3}$

horizontal asymptote is as y = -2/3

Substitute -x for x to test for symmetry to the y-axis

$$f(x) = \frac{2x^2 - 18}{-3x^2 - 12x - 12}$$

$$f(-x) = \frac{2(-x)^2 - 18}{-3(-x)^2 - 12(-x) - 12}$$

$$= \frac{2x^2 - 18}{-3x^2 + 12x - 12}$$

f(-x) is not the same as f(x) so the function is not symmetric to the y-axis.

Exercise 5 (Continued):

Substitute -f(x) for f(x) to test for symmetry to the x-axis

$$
\begin{aligned}
f(x) &= \frac{2x^2 - 18}{-3x^2 - 12x - 12} \\
-f(x) &= -\frac{2x^2 - 18}{-3x^2 - 12x - 12} \\
&= \frac{-2x^2 + 18}{-3x^2 - 12x - 12}
\end{aligned}
$$

-f(x) is not the same as f(x) so the function is not symmetric to the x-axis.

Substitute -x for x and -f(x) for f(x) to test for symmetry to the origin

$$
\begin{aligned}
f(x) &= \frac{2x^2 - 18}{-3x^2 - 12x - 12} \\
-f(-x) &= -\frac{2(-x)^2 - 18}{-3(-x)^2 - 12(-x) - 12} \\
&= -\frac{2x^2 - 18}{-3x^2 + 12x - 12} \\
&= \frac{-2x^2 + 18}{-3x^2 + 12x - 12}
\end{aligned}
$$

-f(-x) is not the same as f(x) so the function is not symmetric to the origin.

Evaluate f(x) at -7 and 7 to get a couple more points for the graph.

$$
\begin{aligned}
f(x) &= \frac{2x^2 - 18}{-3x^2 - 12x - 12} \\
f(-7) &= \frac{2(-7)^2 - 18}{-3(-7)^2 - 12(-7) - 12} \\
&= \frac{80}{-75} \\
&\approx -1.07
\end{aligned}
$$

$$
\begin{aligned}
f(x) &= \frac{2x^2 - 18}{-3x^2 - 12x - 12} \\
f(7) &= \frac{2(7)^2 - 18}{-3(7)^2 - 12(7) - 12} \\
&= \frac{80}{-243} \\
&\approx -0.33
\end{aligned}
$$

Exercise 5 (Continued):

Graph the function using the data found