Review Exercise Set 14

Exercise 1: Write the first five terms of the sequence where \( n \) starts at 1.

\[ a_n = n^2 - 1 \]

Exercise 2: Find the indicated term of the sequence.

\[ a_n = (-1)^n(n - 2)(n + 1) \]

\[ a_{23} = \]

Exercise 3: Write an expression for the \( n^{th} \) term of the given sequence.

\[ \frac{1}{2}, \frac{4}{3}, \frac{9}{4}, \frac{16}{5}, \frac{25}{6}, \ldots \]

Exercise 4: Simplify the following factorial expression.

\[ \frac{8!}{10!} \]
Exercise 5: Write the first five terms of the sequence defined by the recursion formula.

\[ a_1 = 5; \ a_n = a_{n-1} + 4 \]

Exercise 6: Find the sum of the finite series.

\[ \sum_{n=1}^{5} (n^2 - 1) \]
Review Exercise Set 14 Answer Key

Exercise 1: Write the first five terms of the sequence where \( n \) starts at 1.

\[ a_n = n^2 - 1 \]

\[
\begin{align*}
  n = 1 & : a_1 = 1^2 - 1 = 1 - 1 = 0 \\
  n = 2 & : a_2 = 2^2 - 1 = 4 - 1 = 3 \\
  n = 3 & : a_3 = 3^2 - 1 = 9 - 1 = 8 \\
  n = 4 & : a_4 = 4^2 - 1 = 16 - 1 = 15 \\
  n = 5 & : a_5 = 5^2 - 1 = 25 - 1 = 24
\end{align*}
\]

The first five terms of the sequence are 0, 3, 8, 15, 24.

Exercise 2: Find the indicated term of the sequence.

\[ a_n = (-1)^n(n - 2)(n + 1) \]

\[
\begin{align*}
  a_{23} &= (-1)^{23}(23 - 2)(23 + 1) \\
  &= (-1)(21)(24) \\
  &= -504
\end{align*}
\]

Exercise 3: Write an expression for the \( n \)th term of the given sequence.

\[ \frac{1}{2}, -\frac{4}{3}, \frac{9}{4}, -\frac{16}{5}, \frac{25}{6}, \ldots \]

The signs on the terms are alternating between positive and negative so the expression for the \( n \)th term must have \(-1\) raised to an exponent as part of the formula.

\[
(-1)^n \frac{1}{2}, (-1)^n \frac{4}{3}, (-1)^n \frac{9}{4}, (-1)^n \frac{16}{5}, (-1)^n \frac{25}{6}, \ldots
\]

Since the first term is positive the exponent for the first term needs to be an even number. If we let \( n \) start at 1, then this makes our exponent \( n + 1 \).

\[
(-1)^{n+1} \frac{1}{2}, (-1)^{n+1} \frac{4}{3}, (-1)^{n+1} \frac{9}{4}, (-1)^{n+1} \frac{16}{5}, (-1)^{n+1} \frac{25}{6}, \ldots
\]
Exercise 3 (Continued):

The numerators of the fractions are all perfect squares

\[
(-1)^{i+1} \frac{1^2}{i+1}, (-1)^{2+1} \frac{2^2}{3}, (-1)^{3+1} \frac{3^2}{4}, (-1)^{4+1} \frac{4^2}{5}, (-1)^{5+1} \frac{5^2}{6},\ldots
\]

The denominators are all 1 more than the value of \(n\)

\[
(-1)^{i+1} \frac{1^2}{i+1}, (-1)^{2+1} \frac{2^2}{2+1}, (-1)^{3+1} \frac{3^2}{3+1}, (-1)^{4+1} \frac{4^2}{4+1}, (-1)^{5+1} \frac{5^2}{5+1},\ldots
\]

Replacing all of the values that are changing from term to term with \(n\) gives us the expression for the \(n\)th term

\[
a_n = (-1)^{n+1} \frac{n^2}{n+1}
\]

Exercise 4: Simplify the following factorial expression.

\[
\frac{8!}{10!} = \frac{8!}{10 \times 9 \times 8!} = \frac{1}{10 \times 9} = \frac{1}{90}
\]

Exercise 5: Write the first five terms of the sequence defined by the recursion formula.

\[a_1 = 5; \ a_n = a_{n-1} + 4\]

\[
\begin{array}{cccc}
n = 2 & n = 3 & n = 4 & n = 5 \\
a_2 = a_1 + 4 & a_3 = a_2 + 4 & a_4 = a_3 + 4 & a_5 = a_4 + 4 \\
\quad = a_1 + 4 & \quad = a_2 + 4 & \quad = a_3 + 4 & \quad = a_4 + 4 \\
\quad = 5 + 4 & \quad = 9 + 4 & \quad = 13 + 4 & \quad = 17 + 4 \\
\quad = 9 & \quad = 13 & \quad = 17 & \quad = 21 \\
\end{array}
\]

The first five terms of the sequence are 5, 9, 13, 17, and 21.
Exercise 6: Find the sum of the finite series.

\[
\sum_{n=1}^{5} (n^2 - 1) = (1^2 - 1) + (2^2 - 1) + (3^2 - 1) + (4^2 - 1) + (5^2 - 1)
\]

\[
= (1 - 1) + (4 - 1) + (9 - 1) + (16 - 1) + (25 - 1)
\]

\[
= 0 + 3 + 8 + 15 + 24
\]

\[
= 50
\]