Review Exercise Set 14

Exercise 1: Write the first five terms of the sequence where n starts at 1.

$a_n = n^2 - 1$

Exercise 2: Find the indicated term of the sequence.

$a_n = (-1)^n(n - 2)(n + 1)$

$a_{23} =$

Exercise 3: Write an expression for the n^{th} term of the given sequence.

$$\frac{1}{2}, -\frac{4}{3}, \frac{9}{4}, -\frac{16}{5}, \frac{25}{6}, \ldots$$

Exercise 4: Simplify the following factorial expression.

$$\frac{8!}{10!}$$

Exercise 5: Write the first five terms of the sequence defined by the recursion formula.

$a_1 = 5; a_n = a_{n-1} + 4$

Exercise 6: Find the sum of the finite series.

$$\sum_{n=1}^{5}\left(n^2 - 1\right)$$

Review Exercise Set 14 Answer Key

Exercise 1: Write the first five terms of the sequence where n starts at 1.

$a_n = n^2 - 1$

$n=1$	$n=2$	$n=3$	$n=4$	$n=5$
$a_1 = 1^2 - 1$	$a_1 = 2^2 - 1$	$a_1 = 3^2 - 1$	$a_1 = 4^2 - 1$	$a_1 = 5^2 - 1$
$= 1 - 1$	$= 4 - 1$	$= 9 - 1$	$= 16 - 1$	$= 25 - 1$
$= 0$	$= 3$	$= 8$	$= 15$	$= 24$

The first five terms of the sequence are 0, 3, 8, 15, 24.

Exercise 2: Find the indicated term of the sequence.

$a_n = (-1)^n(n - 2)(n + 1)$

$a_{23} = (-1)^{23}(23 - 2)(23 + 1)$
$a_{23} = (-1)(21)(24)$
$a_{23} = -504$

Exercise 3: Write an expression for the n^{th} term of the given sequence.

$$\frac{1}{2}, -\frac{4}{3}, \frac{9}{4}, -\frac{16}{5}, \frac{25}{6}, \ldots$$

The signs on the terms are alternating between positive and negative so the expression for the nth term must have -1 raised to an exponent as part of the formula.

$$(-1)^{?}\frac{1}{2}, (-1)^{?}\frac{4}{3}, (-1)^{?}\frac{9}{4}, (-1)^{?}\frac{16}{5}, (-1)^{?}\frac{25}{6}, \ldots$$

Since the first term is positive the exponent for the first term needs to be an even number. If we let n start at 1, then this makes our exponent n + 1.

$$(-1)^{1+1}\frac{1}{2}, (-1)^{2+1}\frac{4}{3}, (-1)^{3+1}\frac{9}{4}, (-1)^{4+1}\frac{16}{5}, (-1)^{5+1}\frac{25}{6}, \ldots$$

Exercise 3 (Continued):

The numerators of the fractions are all perfect squares

$$(-1)^{1+1}\frac{1^2}{2},(-1)^{2+1}\frac{2^2}{3},(-1)^{3+1}\frac{3^2}{4},(-1)^{4+1}\frac{4^2}{5},(-1)^{5+1}\frac{5^2}{6},\ldots$$

The denominators are all 1 more than the value of n

$$(-1)^{1+1}\frac{1^2}{1+1},(-1)^{2+1}\frac{2^2}{2+1},(-1)^{3+1}\frac{3^2}{3+1},(-1)^{4+1}\frac{4^2}{4+1},(-1)^{5+1}\frac{5^2}{5+1},\ldots$$

Replacing all of the values that are changing from term to term with n gives us the expression for the nth term

$$(-1)^{1+1}\frac{1^2}{1+1},(-1)^{2+1}\frac{2^2}{2+1},(-1)^{3+1}\frac{3^2}{3+1},(-1)^{4+1}\frac{4^2}{4+1},(-1)^{5+1}\frac{5^2}{5+1},\ldots$$

$$a_n=(-1)^{n+1}\frac{n^2}{n+1}$$

Exercise 4: Simplify the following factorial expression.

$$\frac{8!}{10!}=\frac{\cancel{8!}}{10\times 9\times \cancel{8!}}=\frac{1}{10\times 9}=\frac{1}{90}$$

Exercise 5: Write the first five terms of the sequence defined by the recursion formula.

$a_1 = 5$; $a_n = a_{n-1} + 4$

$n=2$ $n=3$ $n=4$ $n=5$

$$\begin{aligned} a_2 &= a_{2-1}+4 \\ &= a_1+4 \\ &= 5+4 \\ &= 9 \end{aligned} \qquad \begin{aligned} a_3 &= a_{3-1}+4 \\ &= a_2+4 \\ &= 9+4 \\ &= 13 \end{aligned} \qquad \begin{aligned} a_4 &= a_{4-1}+4 \\ &= a_3+4 \\ &= 13+4 \\ &= 17 \end{aligned} \qquad \begin{aligned} a_5 &= a_{5-1}+4 \\ &= a_4+4 \\ &= 17+4 \\ &= 21 \end{aligned}$$

The first five terms of the sequence are 5, 9, 13, 17, and 21.

Exercise 6: Find the sum of the finite series.

$$\sum_{n=1}^{5}\left(n^2-1\right)=\left(1^2-1\right)+\left(2^2-1\right)+\left(3^2-1\right)+\left(4^2-1\right)+\left(5^2-1\right)$$
$$=(1-1)+(4-1)+(9-1)+(16-1)+(25-1)$$
$$=0+3+8+15+24$$
$$=50$$