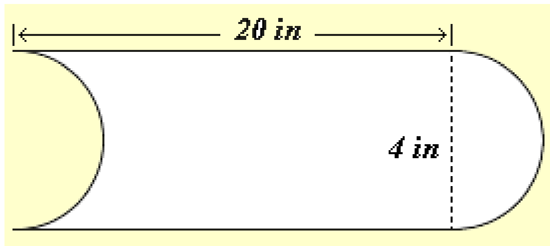
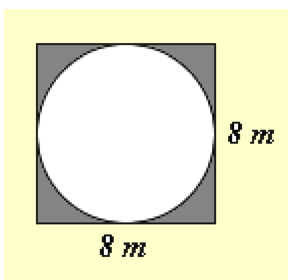
Review Exercise Set 30

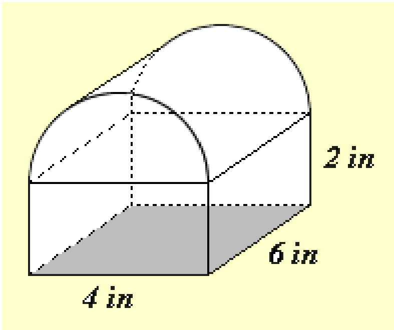
Exercise 1: Find the perimeter of the composite figure below. For calculations involving π give the exact value.



Exercise 2: Find the area of the shaded region for the composite figure below.

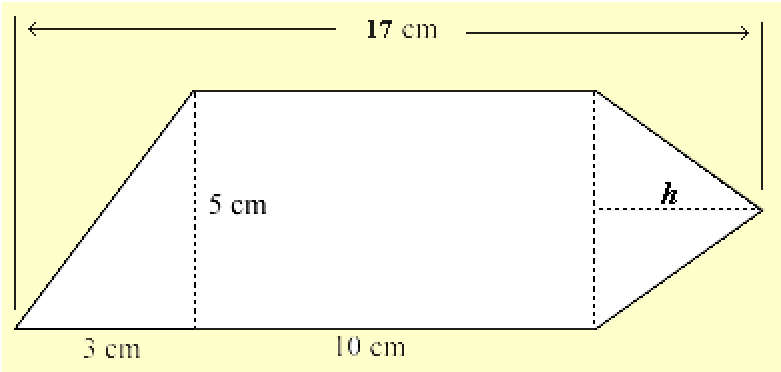


Exercise 3: Find the volume of the composite figure shown below.



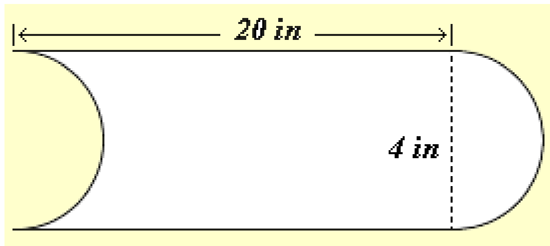
Exercise 4: Find the surface area of the composite figure in Exercise 3.

Exercise 5: Find the area of the composite figure.



Review Exercise Set 30 Answer Key

Exercise 1: Find the perimeter of the composite figure below. For calculations involving π give the exact value.



The semicircle at the right-end of the figure could be moved to the left-end to make the figure a rectangle, whose perimeter would then be twice the length plus twice the width.

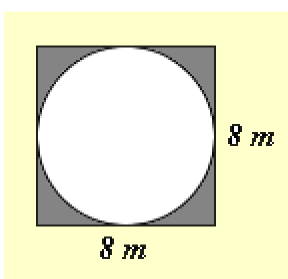
$P = 2L + 2W$

Substitute in the known values for the length and width and evaluate.

$P = 2(20 \text{ in}) + 2(4 \text{ in})$
$P = 40 \text{ in} + 8 \text{ in}$
$P = 48 \text{ in}$

The perimeter of the composite figure is 48 inches.

Exercise 2: Find the area of the shaded region for the composite figure below.



To find the area of the shaded region we must find the area of the outer square and the area of in enclosed circle. The difference between these two areas will leave us with the area of the shaded region.

Area of square (A_s) = s^2

Area of circle (A_c) = πr^2

Exercise 2 (Continued):

Area of shaded region (A) = A_s - A_c

Area of shaded region (A) = $s^2 - \pi r^2$

We know the length of the sides for the square but we need to find the radius of the circle. Since the circle is enclosed within the square the diameter of the circle is equal to the length of the sides of the square. Then taking half of the diameter will give us the radius.

$$d = s$$
$$r = \frac{1}{2} d$$
$$r = \frac{1}{2} s$$
$$r = \frac{1}{2} (8 \text{ m})$$
$$r = 4 \text{ m}$$

Now we can substitute in the values for our variables and evaluate.

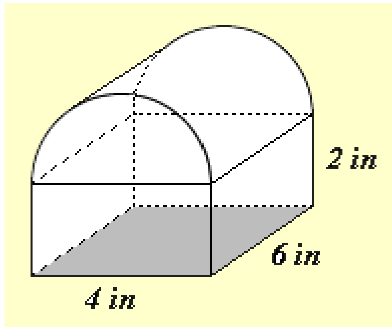
$$A = s^2 - \pi r^2$$
$$A = (8 \text{ m})^2 - \pi (4 \text{ m})^2$$
$$A = (64 \text{ m}^2) - \pi (16 \text{ m}^2)$$
$$A = 64 \text{ m}^2 - 16\pi \text{ m}^2$$
$$A = 64 - 16\pi \text{ m}^2$$

or

$$A \approx 64 - 16(3.14) \text{ m}^2$$
$$A \approx 64 - 50.24 \text{ m}^2$$
$$A \approx 13.76 \text{ m}^2$$

The exact area of the shaded region is 64 - 16π square meters which would be approximately 13.76 square meters.

Exercise 3: Find the volume of the composite figure shown below.



The composite figure consists of half of a right circular cylinder lying on top of a rectangular solid base. The volume of the composite figure would therefore be equal to the sum of the volume of the rectangular base plus half of the volume of a right circular cylinder. Since the cylinder is laying on top of the rectangular base its height would be equal to the length of the rectangular base.

Volume of rectangular base (V_r) = lwh

Volume of a semi right circular cylinder (V_c) = $\frac{1}{2}\pi r^2 h$

Volume of a semi right circular cylinder (V_c) = $\frac{1}{2}\pi r^2 l$

Volume of composite figure (V) = $V_r + V_c$

Volume of composite figure (V) = lwh + $\frac{1}{2}\pi r^2 l$

We know the values of all of our variables except for the radius. However, from the figure we can see that the diameter of the cylinder is equal to the width of the rectangular base (4 in). So the radius must be half of the width (2 in).

d = w

r = $\frac{1}{2}$ d

r = $\frac{1}{2}$ w

r = $\frac{1}{2}$ (4 in)

r = 2 in

Exercise 3 (Continued):

Now substitute the known values and evaluate.

$V = lwh + \frac{1}{2}\pi r^2 l$

$V = (6\text{ in})(4\text{ in})(2\text{ in}) + \frac{1}{2}\pi\ (2\text{ in})^2(6\text{ in})$

$V = (24\text{ in}^3) + \frac{1}{2}\pi\ (4\text{ in}^2)(6\text{ in})$

$V = 24\text{ in}^3 + 12\pi\text{ in}^3$

$V = 24 + 12\pi\text{ in}^3$

or

$V \approx 24 + 12(3.14)\text{ in}^3$

$V \approx 24 + 37.68\text{ in}^3$

$V \approx 61.68\text{ in}^3$

The exact volume of the composite figure is 24 + 12π cubic inches which would be approximately 61.68 cubic inches.

Exercise 4: Find the surface area of the composite figure in Exercise 3.

We would work this problem similar to exercise 3 in that we have the same geometric shapes but now we are looking for the surface area instead of the volume.

Starting with the base, the surface area of a rectangular solid is equal to twice the length times width plus twice the length times height plus twice the width times height.

$SA = 2lw + 2lh + 2wh$

However, since the semi-cylinder is lying on top of the base we would not include the surface area of what would have been the top of the base (length times width) so we must subtract the area of the top from our surface area.

$SA_r = 2lw + 2lh + 2wh - lw$

$SA_r = lw + 2lh + 2wh$

Exercise 4 (Continued):

Now for the cylinder, the surface area of an entire right circular cylinder is equal to two pi times the radius squared plus two pi times the radius times the height. Since we have only half of the cylinder we would take half of the surface area and we will again replace the height with the length of the rectangular base.

$SA = 2\pi r^2 + 2\pi rh$

$SA_c = \frac{1}{2}(2\pi r^2 + 2\pi rl)$

$SA_c = \pi r^2 + \pi rl$

So now we can add our two surface area equations together to obtain the surface area for the composite figure.

$SA = SA_r + SA_c$

$SA = lw + 2lh + 2wh + \pi r^2 + \pi rl$

Now substitute the known values from exercise 3 and evaluate.

l = 6 in, w = 4 in, h = 2 in, and r = 2 in

$SA = (6\text{ in})(4\text{ in}) + 2(6\text{ in})(2\text{ in}) + 2(4\text{ in})(2\text{ in}) + \pi(2\text{ in})^2 + \pi(2\text{ in})(6\text{ in})$

$SA = 24\text{ in}^2 + 24\text{ in}^2 + 16\text{ in}^2 + 4\pi\text{ in}^2 + 12\pi\text{ in}^2$

$SA = 64\text{ in}^2 + 16\pi\text{ in}^2$

$SA = 64 + 16\pi\text{ in}^2$

or

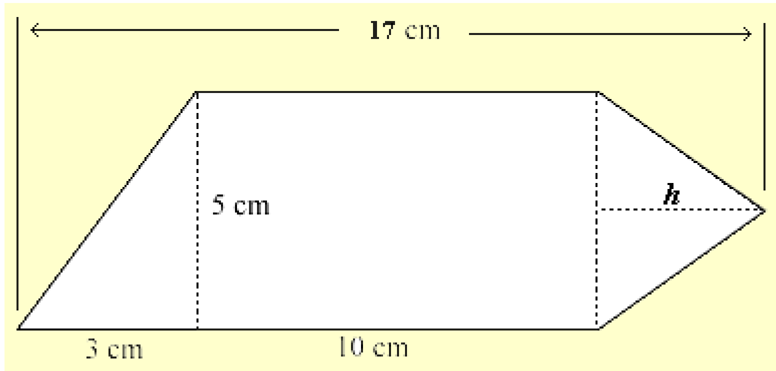
$SA \approx 64 + 16(3.14)\text{ in}^2$

$SA \approx 64 + 50.24\text{ in}^2$

$SA \approx 114.24\text{ in}^2$

The exact surface area of the composite figure is 64 + 16π square inches or approximately 144.24 square inches.

Exercise 5: Find the area of the composite figure.



The composite figure shown above consists of three geometric shapes - two triangles with a rectangle in between them. So in order to find the area of the composite figure we must add up the areas of each individual figure.

Starting from the left, our first shape is a right triangle with a known base and height. The area would be equal to half of the base times height.

$A_1 = \frac{1}{2}bh$

$A_1 = \frac{1}{2}(3\text{ cm})(5\text{ cm})$

$A_1 = 7.5\text{ cm}^2$

The middle shape is the rectangle whose area would be the length times width. The values of the length and width are known so we can substitute them into our equation to find the area of the second shape.

$A_2 = lw$

$A_2 = (10\text{ cm})(5\text{ cm})$

$A_2 = 50\text{ cm}^2$

The third shape is an isosceles triangle with its base against the width of the rectangle. Therefore, the base would be the same as the width. In order to calculate the area of the triangle we need to know its height. The height of the triangle is not given to us directly but we can calculate it by taking the total length of the composite figure and subtract out the length and base of the other two figures.

b = width of rectangle = 5 cm

h = total length - length of rectangle - base of right triangle

h = 17 cm - 10 cm - 3 cm

h = 4 cm

Exercise 5 (Continued):

Now we can find the area of the third shape.

$A_3 = \frac{1}{2}bh$

$A_3 = \frac{1}{2}(5\text{ cm})(4\text{ cm})$

$A_3 = 10\text{ cm}^2$

The total area of the composite figure is the sum of the individual areas so we can now add our individual areas together.

$A = A_1 + A_2 + A_3$

$A = 7.5\text{ cm}^2 + 50\text{ cm}^2 + 10\text{ cm}^2$

$A = 67.5\text{ cm}^2$

The area of the composite figure is 67.5 square centimeters.