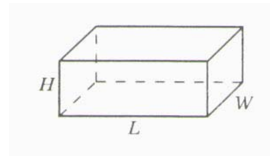
Solids

Objective A:

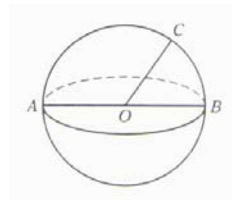
Volume of a Solids

Geometric solids are figures in space. Five common geometric solids are the rectangular solid, the sphere, the cylinder, the cone and the pyramid.

A **rectangular solid** is one in which all sides, called **faces**, are rectangles. The variable *L* is used to represent the length of a rectangular solid, *W* its width, and *H* its height.

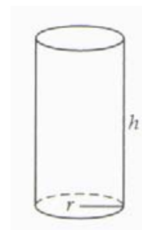


A **sphere** is a solid in which all points are the same distance from point *O*, called the **center** of the sphere. The **diameter**, *d*, of a sphere is a line across sphere going through point *O*. The **radius**, *r*, is a line from the center to a point on the sphere. *AB* is a diameter, and *OC* is a radius of the sphere shown at the right.

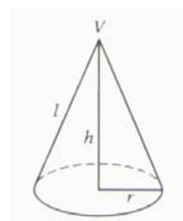


$$d = 2r \quad \text{or} \quad r = \frac{1}{2}d$$

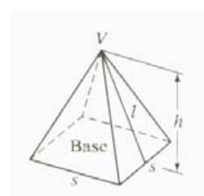
The most common cylinder, called a **right circular cylinder**, is one in which the bases are circles and are perpendicular to the height of the cylinder, and *h* represents the height. We will only discuss right circular cylinders.



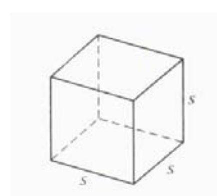
A **right circular cone** is obtained when one base of a circular cylinder is shrunk to a point, called the **vertex**, *V*. The variable *r* is used to represent the radius of the base of the cone, and *h* represents the height. The variable l is used to represent the **slant height**, which is the distance from a point on the circumference of the base to the vertex. We will only discuss right circular cones.



The base of a **regular pyramid** is a regular polygon, and the sides are isosceles triangles. The height, *h*, is the distance to the vertex, *V*, to the base and is perpendicular to the base. The variable l is used to represent the **slant height**, which is the height of one of the isosceles triangles on the face of the pyramid. The regular square pyramid at the right has a square base. This is the only type of pyramids we will discuss.

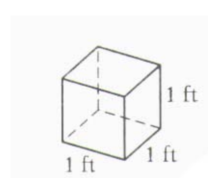


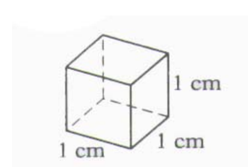
A **cube** is a special type of rectangular solid. Each of the six faces of a cube is a square. The variable *s* is used to represent the length of a side of a cube.



Volume is a measure of the amount of space inside a figure in space. Volume can be use to describe the amount of heating gas used for cooking, the amount of concrete delivered for the foundation of a house, or the amount of water in storage for a city sewer supply.

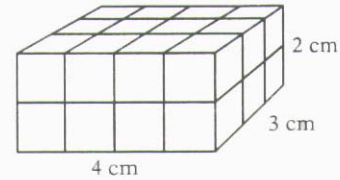
A cube that is 1 foot on each side has a volume of 1 cubic foot, which is written 1 ft^3. A cube that measures 1 cm on each side has a volume of 1 cubic centimeter, written 1 cm^3.





The volume of a solid is the number of cubes that are necessary to exactly fill the solid. The volume of the rectangular solid at the right is 24 cm^3 because it will hold exactly 24 cubes, each 1 cm on a side. Note that the volume can be found by multiplying the length times the width times the height.

$$4 \cdot 3 \cdot 2 = 24$$



Here is a Summary of the formulas for the Geometric Solids

Volumes of Geometric Solids

The volume, *V*, of a **rectangular solid** with length *L*, width *W*, and the height *H* is given by: $V = LWH$.

The volume, *V*, of a **cube** with sides *s* is given by: $V = s^3$.

The volume, *V*, of a **sphere** with radius *r* is given by: $V = \frac{4}{3}\pi r^3$.

The volume, *V*, of a **right circular cylinder** is given by: $V = \pi r^2 h$, where radius *r* of the base *h* is the height.

The volume, *V*, of a **right circular cone** is given by: $V = \frac{1}{3}\pi r^2 h$, where radius *r* of the base *h* is the height.

The volume, *V*, of a **regular square pyramid** is given by: $V = \frac{1}{3}s^2 h$, where s is the length of the side of the base *h* is the height.

Find the volume of a sphere with a diameter of 6 in.

First find the radius of the sphere. $r = \frac{1}{2}d = \frac{1}{2}6 = 3$

Use the formula for the volume of a sphere.

$$V = \frac{4}{3}\pi r^3$$

$$V = \frac{4}{3}\pi (3)^3$$

$$V = \frac{4}{3}\pi (27)$$

The exact volume of the sphere is 36π in^3. $V = 36\pi$

An approximate measure can be found by multiplying 36 by 3.14. $V \approx 113.10$

Example 1

The length of a rectangular solid is 5 m, the width is 3.2m, and the height is 4m. Find the volume of the solid.

Strategy

To find the volume, use the formula for the volume of a rectangular solid. $L = 5$, $W = 3.2$, $H = 4$,

Solution

$V = LWH = 5(3.2)(4) = 64$

The volume of the rectangular solid is 64 m^3.

Example 2

The radius of the base of a cone is 8 cm. The height is 12cm. Find the volume of the cone. Round to the nearest hundredth.

Strategy

To find the volume, use the formula for the volume of a cone. If an approximation is asked for; use $\pi = 3.14$. r = 8, h = 12

Solution

$$V = \frac{1}{3}\pi r^2 h$$

$$V = \frac{1}{3}\pi (8)^2 12 = \frac{1}{3}\pi (64)(12) = V = 265\pi \approx 804.25$$

The volume is approximately 804.25 cm^3.

You Try 1
Find the volume of a cube that measures 2.5 m on a side

Strategy

Solution
15.625 m^3.

You Try 2
The diameter of the base of a cylinder is 8 ft. The height is 22 ft. Find the exact volume of the cylinder.

Strategy

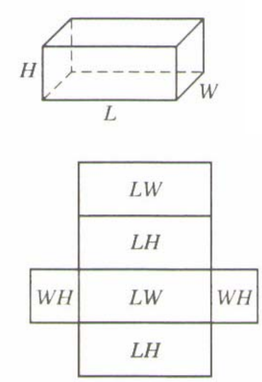
Solution
352π ft^2.

Objective B:

<u>**Surface area of a Solids**</u>

The Surface area of a solid is the total area on the surface of the solid.

When a rectangular is cut open and flattened out, each face is a rectangle. The surface area, *SA*, of the rectangular solid is the sum of the areas of the six rectangles,

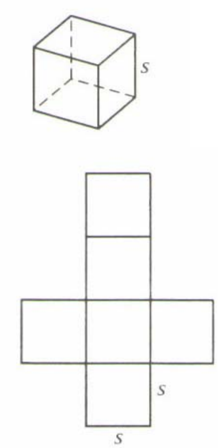


SA = LW + LH + WH + LW + WH + LH

Which simplifies to

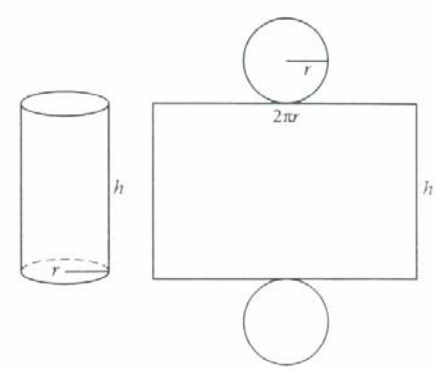
SA = 2LW + 2LH + 2WH

The surface area of a cube is the sum of the areas of six faces of the cube. The area of each face is s^2. Therefore the surface area , SA, of a cube is given by formula SA = 6 s^2.



SA = 6 s^2

When a cylinder is cut open and flattened out, the top and the bottom of the cylinder are circles. The side of the cylinder flattens out to a rectangle. The length of the rectangle is the circumference of the base, which is $2\pi r$; the width is h, the height of the cylinder. Therefore the area of the rectangle is $2\pi rh$. The surface area, SA, of the cylinder is:

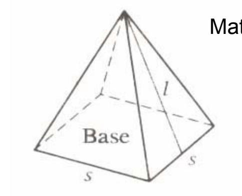


SA = $\pi r^2 + 2\pi rh + \pi r^2$

Which simplifies to

SA = $2\pi r^2 + 2\pi rh$

The surface area of a pyramid is the area of the base plus the area of the four isosceles triangles. A side of the square base is *s*; therefore, the area of the base is s^2. The slant height, *l,* is the height of each triangle, and *s* is the base of each triangle. The surface area, *SA*, of a pyramid is:



$$SA = s^2 + 4\left(\frac{1}{2}sl\right)$$

Which simplifies to

$$SA = s^2 + 2sl$$

Here is a Summary of the formulas for the Surface Area of Geometric Solids

Surface Area of Geometric Solids

The surface area, SA, of a **rectangular solid** with length *L*, width *W*, and the height *H* is given by: SA = 2LW + 2LH + 2WH

The surface area, SA, of a **cube** with sides *s* is given by: $SA = 6s^2$

The surface area, SA, of a **sphere** with radius *r* is given by: $SA = 4\pi r^2$.

The surface area, SA, of a **right circular cylinder** is given by: $SA = 2\pi r^2 + 2\pi rh$, where radius *r* of the base *h* is the height.

The surface area, SA, of a **right circular cone** is given by: $SA = \pi r^2 + \pi rl$, where radius *r* of the base *l* is the slant height.

The surface area, SA, of a **regular square pyramid** is given by: $SA = s^2 + 2sl$, where s is the length of the side of the base *l* is the slant height.

Example 1

Find the surface area of the sphere with a diameter of 18 cm.

Strategy

First find the radius of the sphere.
Use the formula for the surface area of a sphere

Solution

$r = \frac{1}{2}d = \frac{1}{2}(18) = 9$

$SA = 4\pi r^2$.

$SA = 4\pi(9)^2$.

$SA = 4\pi 81$.

$SA = 324\pi$.

$SA \approx 1{,}017.88$

The exact surface area of the sphere is 324π cm^2.
An approximate answer can be found by multiplying 324 by 3.14
The approximate surface area is 1,017.88 cm^2.

Example 2

The diameter of the base of a cone is 5 m. The slant height is 4 m. Find the surface area of the cone. Give the exact measure.

Strategy

To find the surface area of the cone:

1. Find the radius of the base of the cone.
2. Use the formula for the surface area of a cone
   Leave the answer in terms of π.

Solution

$r = \frac{1}{2}d = \frac{1}{2}(5) = 2.5$

$SA = \pi r^2 + \pi rl$

$SA = \pi(2.5)^2 + \pi(2.5)(4)$

$SA = \pi(6.25) + \pi(2.5)(4)$

$SA = 6.25\pi + 10\pi$

$SA = 16.25\pi$

The surface area of the cone is 16.25π m^2

Example 4
Find the area of a label used to cover a soup can that has a radius of 4 cm and a height of 12 cm. Round to the nearest hundredth.
Strategy
To find the area of the label, use the fact that the surface area of the sides of a cylinder is given by $2\pi rh$. An approximation is asked for; use $\pi \approx 3.14$. r = 4, h = 12

Solution
Area of the label $= 2\pi rh$.
Area of the label $= 2(3.14)(4)(12)$

The area is approximately 301.59 cm^2.

You Try 1
The diameter of the base of a cylinder is 6 ft. The height is 8 ft. Find the surface area of the cylinder. Round to the nearest hundredth.

Strategy

Solution
207.35 ft^2.

You Try 2
Which has the larger surface area, a cube with a side measuring 10 cm or a sphere with a diameter measuring 8 cm?

Strategy

Solution
The Cube