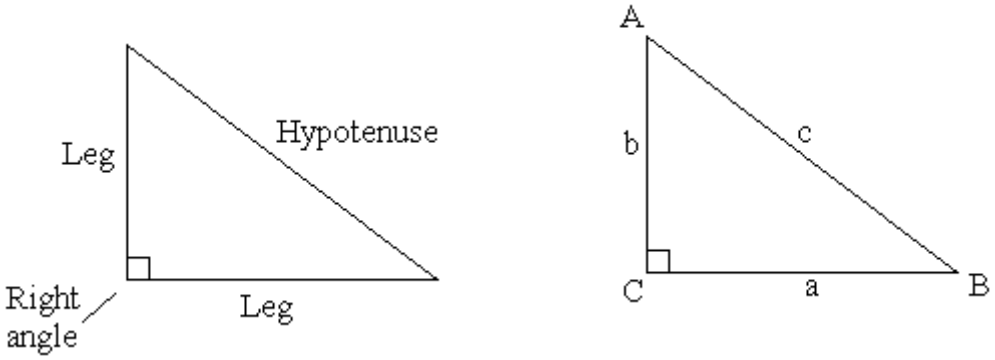
Triangles

The Pythagorean Theorem can be used to find the length of any side of a right triangle as long as the lengths of two of the sides are known.

A right triangle is a triangle that contains one angle with a measurement of 90°, which is referred to as a right angle. The side that is opposite of the right angle is called the hypotenuse (labeled as "c") and the other sides are the legs (labeled "a" and "b").



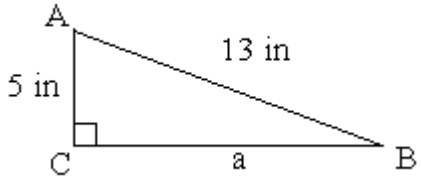
If a and b are the lengths of the legs of a right triangle and c is the length of the hypotenuse, then $c^2 = a^2 + b^2$.

In order to find the length of any side of a right triangle it will be necessary to use the Principal Square Root Property to undo the exponentiation in the Pythagorean Theorem.

The **Principal Square Root** property states that

If $\mathbf{r^2 = s}$, then $\mathbf{r} = \sqrt{s}$ and **r** is the principal square root of **s**.

Example 1. **The length of one leg of a right triangle is 5 in. The hypotenuse is 13 in. Find the length of the other leg.**



Step 1. Substitute the known values into the Pythagorean Theorem

$b = 5, c = 13$ $\qquad c^2 = a^2 + b^2$

$(13)^2 = a^2 + (5)^2$

Step 2. Simplify.

$169 = a^2 + 25$

Step 3. Solve for a.

$169 - 25 = a^2 + 25 - 25$

$144 = a^2$

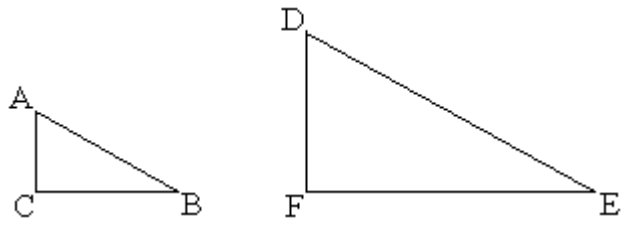
$\sqrt{144} = a$

$12 = a$

The length of the other leg is 12 in.

Objects that have the same shape but not the same size are called similar objects. If two objects are similar then relationships can be made between the sizes of each of the corresponding parts of the objects. Lets look at an example using triangles.

Below are two triangles that have the same shapes but are different sizes.



Knowing that these are similar triangles you can setup the following ratios between the measures of the corresponding sides.

$$\frac{m(\overline{AB})}{m(\overline{DE})} = \frac{m(\overline{BC})}{m(\overline{EF})}\ ,\ \frac{m(\overline{AB})}{m(\overline{DE})} = \frac{m(\overline{AC})}{m(\overline{DF})}\ ,\ \text{and}\ \frac{m(\overline{BC})}{m(\overline{EF})} = \frac{m(\overline{AC})}{m(\overline{DF})}$$

Since the triangles are similar relationships can also be setup between the measures of the corresponding angles.

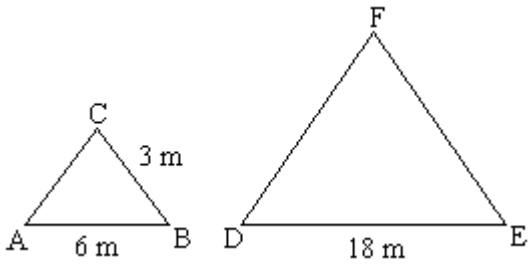
$$m(\angle A) = m(\angle D)\ ,\ m(\angle B) = m(\angle E)\ ,\ \text{and}\ m(\angle C) = m(\angle F)$$

These ratios can be used in solving for the measurements of unknown sides or angles for similar triangles.

Properties of Similar Triangles

1. The ratios of the measures of corresponding sides are equal
2. The measures of the corresponding angles are equal
3. The ratio of the measures of corresponding sides is equal to the ratio of the measures of corresponding heights.

Example 2. **The two triangles below are similar triangles. Find the length of the side EF. Round to the nearest tenth.**



Step 1. **Setup ratios between the corresponding sides of the triangle and substitute the known values.**

$$\frac{m(\overline{DE})}{m(\overline{AB})} = \frac{m(\overline{EF})}{m(\overline{BC})}$$

$$\frac{18}{6} = \frac{m(\overline{EF})}{3}$$

Step 2. **Cross multiply the fractions**

$$18 * 3 = 6 * m(\overline{EF})$$

Step 3. **Solve for the side EF**

$$54 = 6 * m(\overline{EF})$$

$$\frac{54}{6} = \frac{6 * m(\overline{EF})}{6}$$

$$9 = m(\overline{EF})$$

The length of the side EF is 9m.

If two objects have the same shape **AND** the same size, then they are called congruent objects. Similar objects only have to have the same shape.

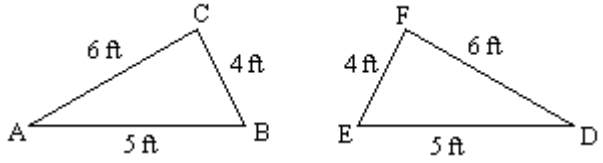
There are three different tests that can be used to determine if two triangles are congruent. The test that is used depends on what information is known about the triangles.

TEST #1: Side-Side-Side Rule (SSS)

As the name implies, this test is used when the measurements for all of the sides of the triangle are known.

Rule: Two triangles are congruent if the measures of the three sides of one triangle equal the measures of the corresponding three sides of the other triangle.

Example 3: Determine if the two triangles are congruent.



Determine which rule to use to test if the triangles are congruent

Since we are given the lengths of all three sides for the two triangles the Side-Side-Side rule will be used.

Determine if the corresponding sides are equal

$m(\overline{AB}) = m(\overline{DE})$ $\quad m(\overline{AC}) = m(\overline{DF})$ $\quad m(\overline{BC}) = m(\overline{EF})$

$5 \text{ ft} = 5\text{ft}$ $\quad 6 \text{ ft} = 6 \text{ ft}$ $\quad 4 \text{ ft} = 4 \text{ ft}$

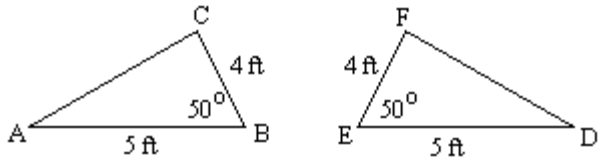
Since the measurements for the three sides of the first triangle are equal to the measurements for the corresponding three sides of the second triangle, these two triangles are congruent by the Side-Side-Side rule.

TEST #2: Side-Angle-Side Rule (SAS)

If the lengths of two sides and the included angle are known for two triangles, this test can be used to test if the triangles are congruent. The included angle is the angle that is created by the two known sides.

Rule: Two triangles are congruent if the measures of the two sides and the included angle of one triangle equal the measures of the corresponding two sides and included angle of the second triangle.

Example 4: Determine if the two triangles are congruent.



Determine which rule to use to test if the triangles are congruent

Since we are given the lengths of two sides and the included angle for the two triangles the Side-Angle-Side rule will be used to test if they are congruent.

Determine if the corresponding sides and angles are equal

$m(\overline{AB}) = m(\overline{DE})$ $m(\overline{BC}) = m(\overline{EF})$ $m(\angle B) = m(\angle E)$

5 ft = 5ft 4 ft = 4 ft 50° = 50°

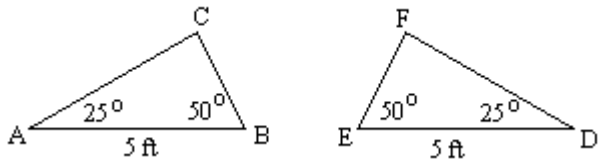
Since the measurements for the two sides and included angle of the first triangle are equal to the measurements for the corresponding two sides and included angle of the second triangle, these two triangles are congruent by the Side-Angle-Side rule.

TEST #3: Angle-Side-Angle Rule (ASA)

This test would be used when the measures of two angles and the lengths of the sides included between these angles are known for two triangles.

Rule: Two triangles are congruent if the measures of two angles and the included side of one triangle equal the measures of the corresponding two angles and included side of the second triangle.

Example 5: Determine if the two triangles are congruent.



Determine which rule to use to test if the triangles are congruent

Since we are given the measurements of two angles and the length of the included side for the two triangles the Angle-Side-Angle rule will be used to test if the triangles are congruent.

Determine if the corresponding sides and angles are equal

$m(\angle A) = m(\angle D)$ $m(\overline{AB}) = m(\overline{DE})$ $m(\angle B) = m(\angle E)$

$25° = 25°$ $5 \text{ ft} = 5 \text{ ft}$ $50° = 50°$

Since the measurements for the two angles and included side of the first triangle are equal to the measurements for the corresponding two angles and included side of the second triangle, these two triangles are congruent by the Angle-Side-Angle rule.