Operations on Rational Numbers

Objective A To add, subtract, multiply, and divide integers

Rules for Addition of Real Numbers:

To add numbers with the same sign, add the absolute values of the numbers. Then attach the sign of the addends.

To add numbers with different signs, find the absolute value of each number. Subtract the smaller of the two numbers from the larger. Then attach the sign of the number with the larger absolute value.

Add: $-65 + (-48)$

The signs are the same. Add the absolute values of the numbers. Then attach the sign of the addends.

$-65 + (-48)$

$|-65| + |-48|$

$65 + 48 = 113$

-113

Add: $27 + (-53)$

$|27| = 27$ and $|-53| = 53$ — The signs are different. Find the absolute value of each number.

$53 - 27 = 26$ — Subtract the smaller number from the larger.

$27 + (-53) = -26$ — Because the $|-53| > |-27|$, attach the sign of -53.

Subtraction is defined as addition of the additive inverse.

Rule for Subtraction of Real Numbers:

If a and b are real numbers, then $a - b = a + (-b)$.

Subtract: $48 - (-22)$

$48 - (-22) = 48 + 22 = 70$ — Change the subtraction minus sign to the addition plus sign. The negative of a negative 22 is then a positive 22.

Subtract: – 31 – 18

– 31 – 18 = – 31 + (– 18) = – 49 Change the subtraction minus sign to a to an addition plus sign. The negative of a positive 18 is a negative 18.

Simplify: – 3 – (– 16) + (– 12)

– 3 – (– 16) + (– 12) = – 3 + 16 + (– 12) Write subtraction as addition of the opposite.

= 13 + (– 12) = 1 Add from left to right.

Rules for Multiplication of Real Numbers:

The **product** of two numbers with the same sign is positive.

The **product** of two numbers with different signs is negative.

Multiply: – 4 (– 9)

– 4 (– 9) = 36 The product of two numbers with the same sign is positive.

Multiply: 84 (– 4)

84 (– 4) = – 336 The product of two numbers with different signs is negative.

The **multiplicative inverse** of a nonzero real number a is $\frac{1}{a}$. This number is also called the reciprocal of a. For instance, the reciprocal of 2 is $\frac{1}{2}$ and the reciprocal of $-\frac{3}{4}$ is $-\frac{4}{3}$. Division of a real number is defined in terms of multiplication of the multiplicative inverse.

Rule for Division of Real Numbers:

If a and b are real numbers and $b \neq 0$, then $a \div b = a \cdot \frac{1}{b}$

Because division is defined in terms of multiplication, the sign rules for dividing real numbers are the same as for multiplying.

Divide: $\frac{-54}{9}$

$\frac{-54}{9} = -6$ The quotient of two numbers with different signs is negative.

Divide: (– 21) ÷ (– 7)

(– 21) ÷ (– 7) = 3 The quotient of two numbers with the same sign is positive

If *a* and *b* are real numbers and *b* ≠ 0, then $\frac{-a}{b} = \frac{a}{-b} = -\frac{a}{b}$.

Properties of Zero and One in Division:

- Zero divided by any number other than zero is zero. $\frac{0}{a} = 0,\ a \neq 0$.
- Division by zero is not defined.
- Any number other than zero divided by itself is 1. $\frac{a}{a} = 1, a \neq 0$
- Any number divided by 1 is the number. $\frac{a}{1} = a$

<u>Objective B</u> <u>To add, subtract, multiply, and divide rational numbers</u>

Recall that a rational number is one that can be written in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$. Examples of rational numbers are $-\frac{5}{9}$ and $\frac{12}{5}$. The number $\frac{9}{\sqrt{7}}$ is not a rational number because $\sqrt{7}$ is not an integer. All integers are rational numbers. Terminating and repeating decimals are also rational numbers.

Add: – 12.34 + 9.059

12.340 – 9.059 = 3.281 The signs are different. Subtract the absolute values of the numbers.

-12.34 + 9.059 = -3.281 Attach the sign of the number with the larger absolute value.

Multiply: (– 0.23)(0.04)

(-0.23)(0.04) = -0.0092 The signs are different. The product is negative.

Divide: (– 4.0764) ÷ (-1.72)

(– 4.0764) ÷ (-1.72) = 2.37 The signs are same. The quotient is positive.

To add or subtract rational numbers written as fractions, first rewrite the fractions as equivalent fractions with a common denominator. The common denominator is the least common multiple (LCM) of the denominators.

Add: $\frac{5}{6}+\left(-\frac{7}{8}\right)$

$\frac{5}{6}+\left(-\frac{7}{8}\right)=\frac{5}{6}\cdot\frac{4}{4}+\left(\frac{-7}{8}\cdot\frac{3}{3}\right)$ The common denominator is 24. Write each factor in terms of the common denominator.

$=\frac{20}{24}+\frac{-21}{24}=\frac{20-21}{24}$ Add the numerators. Place the sum over a common denominator.

$=\frac{-1}{24}=-\frac{1}{24}$

Objective C To evaluate exponential expressions

Repeated multiplication of the same factor can be written using an exponent.

$2*2*2*2*2*2=2^6$ ← Exponent; ↑ Base

$b*b*b*b*b*b=b^6$ ← Exponent; ↑ Base

The **exponent** indicates how many times the factor, called the **base**, occurs in the multiplication. The multiplication of $2\cdot2\cdot2\cdot2\cdot2\cdot2$ is in **factored form**. The exponential expression 2^6 is in the **exponential form**.

2^1 is read "the first power of two" or just "two."
2^2 is read "the second power of two" or just "two squared."
2^3 is read "the third power of two" or just "two cubed."
2^4 is read "the fourth power of two."
2^5 is read "the fifth power of two."
b^5 is read "the fifth power of *b*."

n*th Power of *a

If *a* is a real number and *n* is a positive integer, the nth power of *a* is the product of *n* factors of *a*.

$$a^n = \underbrace{a \cdot a \cdot a \cdot \cdots \cdot a}_{a \text{ as a factor } n \text{ times}}$$

$$5^3 = 5 * 5 * 5 = 125$$
$$(-3)^4 = (-3)(-3)(-3)(-3) = 81$$
$$-3^4 = -(3)^4 = -(3 \cdot 3 \cdot 3 \cdot 3) = -81$$

Note the difference between $(-3)^4$ and -3^4. The placement of the parentheses is very important.

<u>Objective D</u> <u>To use the Order of Operation Agreement</u>

Suppose we wish to evaluate 16 + 4 · 2. There are two operations, addition and multiplication. The operations could be performed in different orders.

Multiply first. $16 + \underbrace{4 \cdot 2}$ or Add first. $\underbrace{16 + 4} \cdot 2$

Then add. $\underbrace{16 + 8}$ Then multiply. $\underbrace{20 \cdot 2}$

24 40

$$24 \neq 40$$

Note that the answers are different. To avoid possibly getting more than one answer to the same problem, an Order of Operation is followed.

Order of Operations Agreement:

Step 1 Perform operations inside grouping symbols. Grouping symbols include parentheses ( ), brackets [ ], braces { }, the absolute value symbol, and the fraction bar.
Step 2 Simplify exponential expressions.
Step 3 Do multiplication and division as they occur from left to right.
Step 4 Do addition and subtraction as they occur from left to right.

Simplify: $8-\frac{2-22}{4+1}\cdot 2^2$

$$8-\frac{2-22}{4+1}\cdot 2^2$$

$$=8-\frac{-20}{5}\cdot 2^2$$

The fraction bar is a grouping symbol. Perform the operations above and below the fraction bar.

$$=8-\frac{-20}{5}\cdot 4$$

Simplify the exponential expressions.

$$=8-(-4)\cdot 4$$
$$=8-(-16)$$

Do multiplication and division as they occur from left to right.

$$=24$$

Do addition and subtraction as they occur from left to right.

Simplify: $14-[(25-9)\div 2]^2$

$$14-[(25-9)\div 2]^2$$
$$=14-[(16)\div 2]^2$$
$$=14-[8]^2$$
$$=14-64$$
$$=-50$$

A complex fraction is a fraction whose numerator or denominator contains one or more fractions. Examples of complex fractions are shown below.

For example $\dfrac{\frac{2}{3}}{-\frac{5}{2}}$ and $\dfrac{\frac{1}{7}+5}{\frac{3}{4}-\frac{7}{8}}$.

When simplifying complex fractions, recall that $\dfrac{\frac{a}{b}}{\frac{c}{d}} = \dfrac{a}{b} \div \dfrac{c}{d} = \dfrac{a}{b} \cdot \dfrac{d}{c}$.

Simplify: $\dfrac{\frac{3}{4} - \frac{1}{3}}{\frac{1}{5} - 2}$

$$\dfrac{\frac{3}{4} - \frac{1}{3}}{\frac{1}{5} - 2} = \dfrac{\frac{9}{12} - \frac{4}{12}}{\frac{1}{5} - \frac{10}{5}} = \dfrac{\frac{5}{12}}{-\frac{9}{5}}$$

Perform the operations above and below the main fraction bar.

$$\dfrac{\frac{5}{12}}{-\frac{9}{5}} = \dfrac{5}{12}\left(-\dfrac{5}{9}\right) = -\dfrac{25}{108}$$

Multiply the numerator of the complex fraction by the reciprocal of the complex denominator.