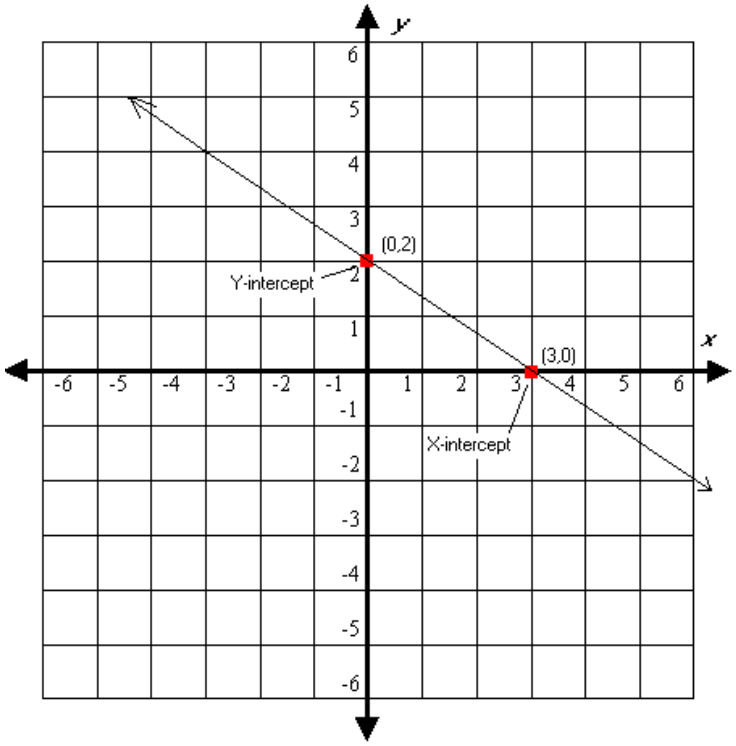
INTERCEPTS AND SLOPES OF STRAIGHT LINES

♦ **To find the x- and y-intercepts of a straight line**

Example 1

Find the x-intercept and the y-intercept of the graph of the equation 2x + 3y = 6

The graph of the equation **2x + 3y = 6** is shown below. The **graph crosses the x-axis** at the **point (3, 0).** This point is called the **x-intercept**. The **graph also crosses the y-axis** at the **point (0, 2)**. This point is called the **y-intercept**.



To find the x-intercept, let y = 0.

(Any point on the x-axis has y-coordinate 0)

$2x + 3y = 6$

$2x + 3(0) = 6$

$2x = 6$

$\mathbf{x = 3}$

The x-intercept is (3, 0)

To find the y-intercept, let x = 0.

(Any point on the y-axis has x-coordinate 0)

$2x + 3y = 6$

$2(0) + 3y = 6$

$3y = 6$

$\mathbf{y = 2}$

The y-intercept is (0, 2)

For any equation of the form **y = mx + b**, the **y-intercept is (0, b)**

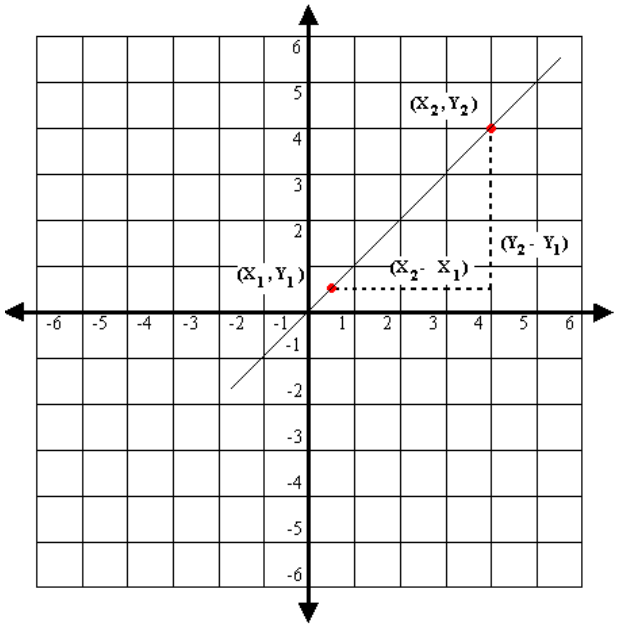
Example 2

Find the y-intercept of y = 3x + 4

y = 3x + 4 = 3(0) + 4 = 4 **(let x = 0)**

The y-intercept is (0, 4)

- **To find the slope of a straight line**



Slope Formula

If $\mathbf{P_1(x_1, y_1)}$ and $\mathbf{P_2(x_2, y_2)}$ are two points on a line and $\mathbf{x_1 \neq x_2}$, then $\mathbf{m = \frac{y_2 - y_1}{x_2 - x_1}}$

If $\mathbf{x_1 = x_2}$, the **slope** is **undefined**.

Example 3

Find the slope of the line containing the points (-1, 1) and (2, 3)

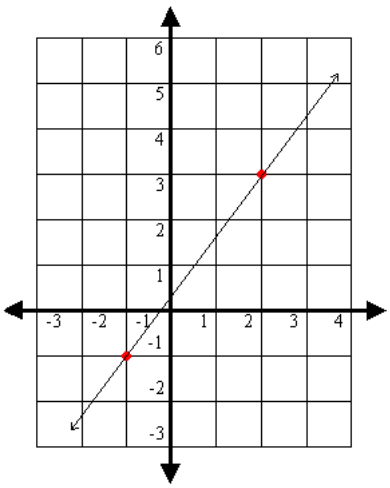
Let P_1 be (-1, 1) and P_2 be (2, 3)

Then,

$X_1 = -1$, $Y_1 = 1$, $X_2 = 2$, $Y_2 = 3$.

$$m = \frac{Y_2 - Y_1}{X_2 - X_1} = \frac{3-1}{2-(-1)} = \frac{2}{3}$$

The slope is $\frac{2}{3}$



A line that slants **upward to the right** always has a **positive slope.**

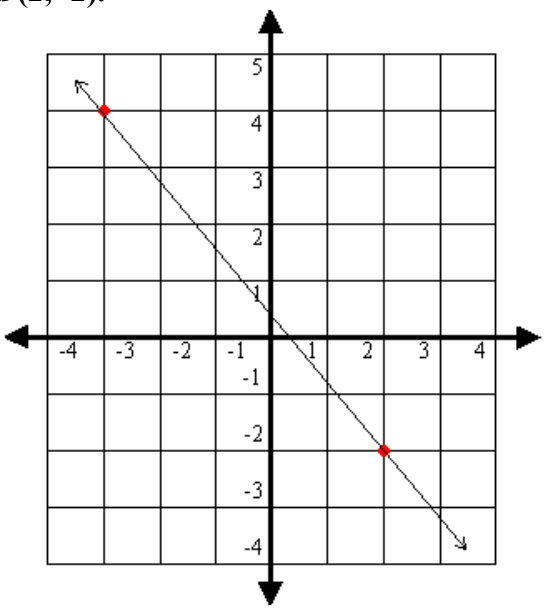
Example 4

Find the slope of the line containing the points (-3, 4) and (2, -2).

Let P_1 be (-3, 4) and P_2 be (2, -2).

$$m = \frac{Y_2 - Y_1}{X_2 - X_1} = \frac{-2-4}{2-(-3)} = \frac{-6}{5} = -\frac{6}{5}$$

The slope is $-\frac{6}{5}$



A **line** that slants **downward to the right** always has a **negative slope**.

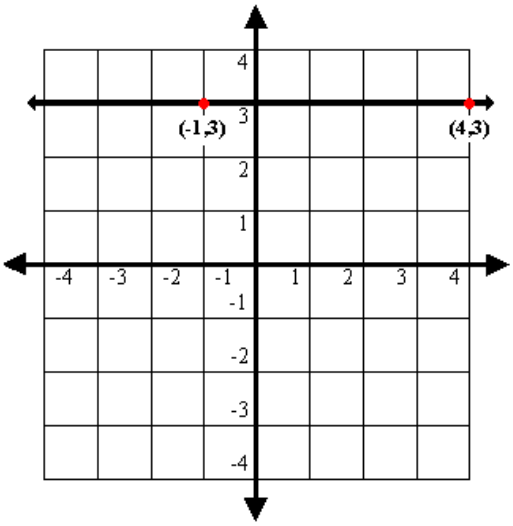
Example 5

Find the slope of the line containing the points (-1, 3) and (4, 3).

Let P_1 be (-1, 3) and P_2 be (4, 3).

$$m = \frac{Y_2 - Y_1}{X_2 - X_1} = \frac{3-3}{4-(-1)} = \frac{0}{5} = 0$$

The slope is 0



A **horizontal line** has **zero slope.**

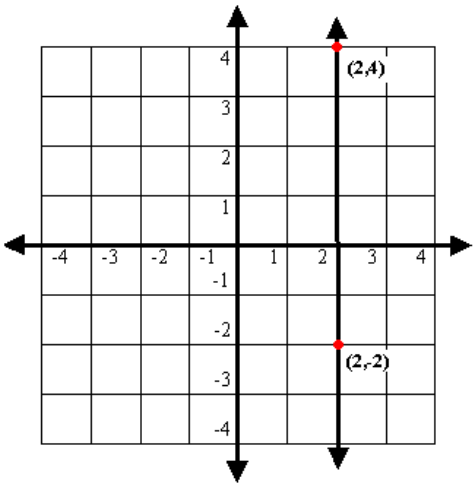
Example 6

Find the slope of the line containing the points (2, -2) and (2, 4).

Let P1 be (2, -2) and P2 be (2, 4).

$$m = \frac{Y_2 - Y_1}{X_2 - X_1} = \frac{4-(-2)}{2-2} = \frac{6}{0}$$

The slope is undefined.



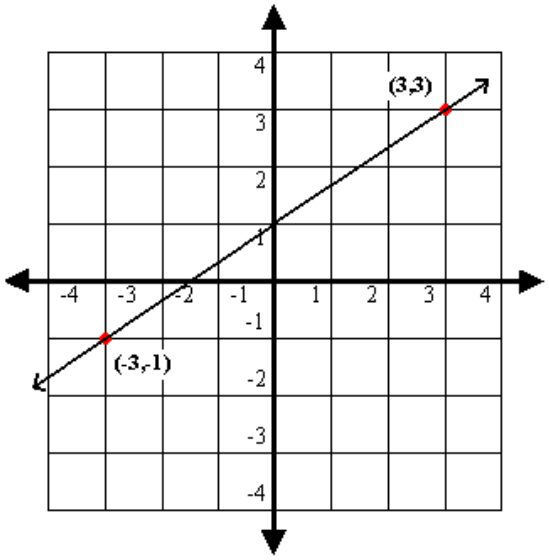
- **To graph a line using the slope and the y-intercept**

Slope-Intercept Equation of a line

An equation of the form **y = mx + b** is called **slope-intercept** form of a straight line. The slope of the line is **m**, the **coefficient of x**. The **y-intercept is (0, b),** where **b** is the **constant term of the equation**.

The graph of the equation $y = \frac{2}{3}x + 1$ is shown at the right. The points **(-3, -1)** and **(3, 3)** are on the graph. The slope of the line between the two points is $m = \frac{3-(-1)}{3-(-3)} = \frac{4}{6} = \frac{2}{3}$.

Observe that the slope of the line is the coefficient of x in the equation $y = \frac{2}{3}x + 1$**.** Also recall that the **y-intercept** is **(0, 1),** where 1 is the constant term of the equation.



The following equations are written in slope-intercept form.

y = 2x – 3	Slope = 2; y-intercept = (0, -3)
y = -x + 2	Slope = -1 (-x = -1 x); y-intercept = (0, 2)
$y = \frac{x}{2}$	Slope = $\frac{1}{2}\left(\frac{x}{2} = \frac{1}{2}x\right)$; y-intercept = (0, 0)

Example 7

Graph y = 2x – 3

y-intercept = (0, b) = (0, -3)

$m = 2 = \frac{2}{1} = \frac{\text{Change in y}}{\text{Change in x}}$

Beginning at the y-intercept, move right 1 unit (change in x) and then up 2 units (change in y).

(1, -1) is a second point on the graph.

Draw a line through the two points **(0, -3)** and **(1, -1).**