Addition and Subtraction of Polynomials

A **monomial** is a number, a variable, or a product of numbers and variables. The following examples are monomials. The degree of a monomial is the sum of the exponents of the variables.

x	Degree = 1 $(x = x^1)$
$3y^2$	Degree = 2
$-4a^2b$	Degree = 3
$2x^2yz^3$	Degree = 6

The degree of a nonzero constant is zero.
Ex: the degree of 7 is 0

Each of the addends of a variable expression is called a term. For example, the following variable expression:

$$4x - 3xy + 4z^2$$

The terms are $4x$, $-3xy$, and $4z^2$. Note that to determine the terms of an expression, subtraction is re-written as addition of the opposite.

A **polynomial** is a variable expression in which the terms are monomials.

A polynomial of *one* term is a **monomial**	Example:	$-7x^2$
A polynomial of *two* terms is a **binomial**	Example:	$4x + 2$
A polynomial of *three* terms is a **trinomial**	Example:	$7x^2 + 5x - 7$
A polynomial of *four or more* terms is simply called a **polynomial**	Example :	$3x^3 - 4x^2 + 2x - 8$

The terms of a polynomial in one variable are usually arranged so that the exponents of the variable decrease from left to right. This is called **descending order**.

$5x^3 - 4x^2 + 6x - 1$
$7z^5 + 4z^3 + z^2 - 6$
$2y^4 + y^3 - 2y^2 + 4y - 1$

The **degree** of a polynomial is the degree of the term of largest degree.

The degree of $\mathbf{4x^3 - 5x^2 + 7x - 8}$ is 3.

The degree of $\mathbf{3x^3y - 4xy^2 + 2xy}$ is 4

Polynomials can be added, using either a horizontal or vertical format, by combining like terms.

Simplify: $(3x^3 - 7x + 2) + (7x^2 + 2x - 7)$. Use a horizontal format.

$(3x^3 - 7x + 2) + (7x^2 + 2x - 7)$

☞ Use the commutative and Associative properties of Addition to re-arrange and group like terms.

$= 3x^3 + 7x^2 + (-7x + 2x) + (2-7)$

$= 3x^3 + 7x^2 - 5x - 5$ ☞ Then combine like terms.

Simplify: $(-4x^2 + 6x - 9) + (12 - 8x + 2x^3)$. Use a vertical format.

$$\begin{array}{r} -4x^2 + 6x - 9 \\ 2x^3 \qquad\quad - 8x + 12 \\ \hline 2x^3 - 4x^2 - 2x + 3 \end{array}$$

☞ Arrange the terms of each polynomial in descending order with like terms in the same column

☞ Combine the terms in each column

Subtraction of polynomials

The **opposite** of the polynomial $(3x^2 - 7x + 8)$ is $-(3x^2 - 7x + 8)$
To simplify the opposite of a polynomial, change the sign of each term inside of the parentheses.

Polynomials can be subtracted using either a horizontal or vertical format. To subtract, add the opposite of the second polynomial to the first.

Simplify: $(4y^2 - 6y + 7) - (2y^3 - 5y - 4)$. **Use a horizontal format.**

$(4y^2 - 6y + 7) - (2y^3 - 5y - 4)$

$= (4y^2 - 6y + 7) + (-2y^3 + 5y + 4)$ ☞ Add the opposite of the second polynomial to the first

$= -2y^3 + 4y^2 + (-6y + 5y) + (7 + 4)$

$= -2y^3 + 4y^2 - y + 11$ ☞ Combine like terms

Simplify: $(9 + 4y + 3y^3) – (2y^2 + 4y – 21)$. **Use a vertical format**.

The opposite of $2y^2 + 4y – 21$ is $–2y^2 – 4y + 21$

$$\begin{array}{rrrr} 3y^3 & & +\,4y & +\;9 \\ & -2y^2 & -\,4y & +\,21 \\ \hline 3y^3 & -\,2y^2 & & +\,30 \end{array}$$

☞ Arrange the terms of each polynomial in descending order with like terms in the same column

☞ Note that $4y – 4y = 0$, but 0 is not written.