INTEGER EXPONENTS AND SCIENTIFIC NOTATION

Objective A: To Divide Monomials

Rules:

1. To multiply two monomials with the same base, add the exponents.

   EX: \( X^7 \cdot X^5 = X^{7+5} = X^{12} \)
   
   \((-2x)(3x^2)\)
   
   EX: \(-6x^{1+2}\)
   
   \(-6x^3\)

2. To divide two monomials with the same base, subtract the exponents of the like bases:

   EX: Simplify
   
   \( \frac{a^7}{a^4} \)
   
   \( \frac{a^7}{a^4} = a^{7-4} = a^3 \)

   Simplify
   
   \( \frac{r^7n^5}{r^4n^2} \)
   
   \( \frac{r^7n^5}{r^4n^2} = r^{7-4}n^{5-2} = r^3n^3 \)

3. When the exponent equals to zero, as long as the base is not zero, then the result will be one.

   \( \frac{x^4}{x^4} = x^{4-4} = x^0 = 1 \quad x \neq 0 \)
Simplify \((12a^3)^0\); \(a \neq 0\)

\[(12a^3)^0 = 1\]

Simplify \(- (4x^8y^3)^0\); \(x \neq 0\) and \(y \neq 0\)

\[-(4x^8y^3)^0 = -1\]

4. Definition of a Negative Exponent: If \(x \neq 0\) and \(n\) is a positive integer, then:

\[x^{-n} = \frac{1}{x^n}\] and \[\frac{1}{x^{-n}} = x^n\]

Evaluate: \(2^{-4}\)

\[2^{-4} = \frac{1}{2^4} = \frac{1}{16}\]

5. Rule for Simplifying Powers of Quotients. If \(m, n,\) and \(Q\) are integers and \(y \neq 0\), then

\[
\left( \frac{x^m}{y^n} \right)^p = \frac{x^{mp}}{y^{np}}
\]

Evaluate: \(\left( \frac{x^3}{y^5} \right)^2\)

\[
\left( \frac{x^3}{y^5} \right)^2 = \frac{x^{3(2)}}{y^{5(2)}} = \frac{x^6}{y^{10}}
\]
6. Rule for Negative Exponents on Fraction Expression: If $a \neq 0$, $b \neq 0$, and $n$ is a positive integer, then

\[
\left( \frac{a}{b} \right)^{-n} = \left( \frac{b}{a} \right)^n
\]

Simplify:

\[
\left( \frac{2x^3}{b^4} \right)^{-2} = \left( \frac{b^4}{2x^3} \right)^{2} = \frac{b^{4(2)}}{2^2x^{3(2)}} = \frac{b^8}{4x^6}
\]

**Objective B: To write a Number in Scientific Notation**

Due the fact that in science, we have to deal with very small or very large numbers, for simplistic matter of reading, we rewrite them by using scientific notation. In this notation, a number is expressed as the product of two factors, one between 0 and 10, and the other a power of 10.

To write in scientific notation, write it in form

\[a \times 10^n\]

where $a$ is a number between 1 and 10, and $n$ is an integer.

1. Converting Ordinary notation into Scientific Notation.

   For numbers greater than or equal to 10, move the decimal point to the right of the first digit. The exponent $n$ is positive and equal to the number of places the decimal point has been moved.

   Ex: $240,000 = 2.4 \times 10^5$

   For numbers less than 1, move the decimal point to the right of the first nonzero digit. The exponent $n$ is negative. The absolute value of the exponent is equal to the number of places the decimal point has been moved.

   Ex: $0.000325 = 3.25 \times 10^{-4}$
2. Converting Scientific Notation into Scientific Notation.

When the exponent is positive, move the decimal point to the right the same number of places as the exponents.

EX: $3.45\times10^6 = 3,450,000$.

When the exponents is negative, move the decimal point to the left the same number of places as the absolute value of the exponent.

EX: $8.12\times10^{-3} = 0.00812$