Multiplication of Polynomials

Objective A To multiply a polynomial by a monomial

To multiply a polynomial by a monomial, use the Distribution Property and the Rule for Multiplying Exponential Expressions.

Simplify: $\mathbf{-3a(4a^2 - 5a - 6) = -3a\,(4a^2) - (-3a * 5a) - (-3a * 6)}$
$\mathbf{= -12a^3 - (-15a^2) - (-18a)}$
$\mathbf{= -12a^3 + 15a^2 + 18a}$

Example 1

Simplify: $(5x + 4)(-2x)$

Solution: $(5x + 4)(-2x) = 5x(-2x) + 4(-2x)$
$= -10x^2 - 8x$

Example 2

Simplify: $2a^2b(4a^2 - 2ab + b^2)$

Solution:

$$2a^2b(4a^2 - 2ab + b^2) = 2a^2b(4a^2) - 2a^2b(2ab) + 2a^2b(b^2)$$
$$= 8a^4b - 4a^3b^2 + 2a^2b^3$$

Objective B To multiply two polynomials

Multiplication of two polynomials requires the repeated application of the Distributive Property.

$$(y - 2)(y^2 + 3y + 1) = (y - 2)(y^2) + (y - 2)(3y) + (y - 2)(1)$$
$$= y^3 - 2y^2 + 3y^2 - 6y + y - 2$$
$$= y^3 + y^2 - 5y - 2$$

A convenient method of multiplying two polynomials is to use a vertical format similar to that used for multiplication of whole numbers.

Example of using the vertical format to perform the multiplication of polynomials

$$
\begin{array}{rl}
y^2 + 3y + 1 & \\
y - 2 & \\
\hline
-2y^2 - 6y - 2 & \text{[Multiply } y^2 + 3y + 1 \text{ by } -2] \\
y^3 + 3y^2 + y \quad\quad & \text{[Multiply } y^2 + 3y + 1 \text{ by } y] \\
\hline
y^3 + y^2 - 5y - 2 & \text{[Add the terms in each column]}
\end{array}
$$

The resulting answer is the same under both formats.

Simplify: **$(2a^3 + a - 3) (a + 5)$**

$$
\begin{array}{r}
2a^3 + 0a^2 + a - 3 \\
a + 5 \\
\hline
10a^3 + 0a^2 + 5a - 15 \\
2a^4 + 0a^3 + a^2 - 3a \quad\quad\quad \\
\hline
2a^4 + 10a^3 + a^2 + 2a - 15
\end{array}
$$

Objective C To multiply two binomials

It is frequently necessary to find the product of two binomials. The product can be found using a method called **FOIL**, which is based on the Distributive Property. The letters of **FOIL** stand for **F**irst, **O**utside, **I**nside, **L**ast.

Simplify: $(2x + 3)(x + 5)$

Multiply the First terms.	$(2x + 3)(x + 5)$	$2x * x = 2x^2$
Multiply the Outer terms.	$(2x + 3)(x + 5)$	$2x * 5 = 10x$
Multiply the Inner terms.	$(2x + 3)(x + 5)$	$3 * x = 3x$
Multiply the Last terms.	$(2x + 3)(x + 5)$	$3 * 5 = 15$

Add the combined terms.

$$(2x + 3)(x + 5) = \underset{\mathbf{F}}{2x^2} + \underset{\mathbf{O}}{10x} + \underset{\mathbf{I}}{3x} + \underset{\mathbf{L}}{15} = 2x^2 + 13x + 15$$

Simplify: $(4x - 3)(3x - 2)$

$$\begin{aligned}
& \qquad\quad \textbf{F} \qquad\quad \textbf{O} \qquad\quad \textbf{I} \qquad\qquad \textbf{L} \\
(4x - 3)(3x - 2) &= 4x(3x) + 4x(-2) + (-3)(3x) + (-3)(-2) \\
&= 12x^2 - 8x - 9x + 6 \\
&= 12x^2 - 17x + 6
\end{aligned}$$

Simplify: $(3x - 2y)(x + 4y)$

$$\begin{aligned}
& \qquad \textbf{F} \qquad\quad \textbf{O} \qquad\qquad \textbf{I} \qquad\qquad \textbf{L} \\
(3x - 2y)(x + 4y) &= 3x(x) + 3x(4y) + (-2y)(x) + (-2y)(4y) \\
&= 3x^2 + 12xy - 2xy - 8y^2 \\
&= 3x^2 + 10xy - 8y^2
\end{aligned}$$

Objective D: To multiply binomials that have a special products

Using FOIL, it is possible to find a pattern for the product of the sum and difference of two terms and for the square of a binomial.

The Sum and Difference of Two Terms

$$\begin{aligned}
(a + b)(a - b) &= a^2 + ab - ab - b^2 \\
&= a^2 - b^2
\end{aligned}$$

The Square of a Binomial

$$\begin{aligned}
(a + b)^2 = (a + b)(a + b) &= a^2 + ab + ab + b^2 \\
&= a^2 + 2ab + b^2
\end{aligned}$$

Simplify: $(2x + 3)(2x - 3)$

$$\begin{aligned}
(2x + 3)(2x - 3) &= (2x)^2 - 3^2 \\
&= 4x^2 - 9
\end{aligned}$$

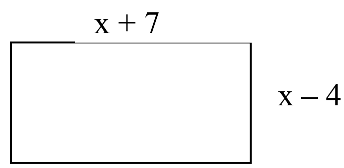
Simplify: $(3x - 2)^2$ (This is the square of a binomial)

$$\begin{aligned}
(3x - 2)^2 &= (3x)^2 + 2(3x)(-2) + (-2)^2 \\
&= 9x^2 - 12x + 4
\end{aligned}$$

Objective E: To solve application problems

Example 3

The length of a rectangle is (x + 7) m. The width is (x – 4) m. Find the area of the rectangle in terms of variable x.



Strategy

To find the area, replace the variable L and W in the equation A = L * W by the given values and solve for A

Solution

A = L * W
$A = (x + 7)(x - 4)$
$A = x^2 - 4x + 7x - 28$
$A = x^2 + 3x - 28$

The area is $(x^2 + 3x - 28)$ m^2.