Ratio and Proportion

Objective A: Solving Proportions

Quantities are groups of units such as 10 feet, 20 ounces and 4 hours. Each part of the group; such as 1 foot, 1 ounce or 1 hour; is a unit of the group called a quantity.

Ratios are the quotients of two quantities. In other words, they are the result of dividing one quantity by another where the units in the quantities are of the same kind.

Example:

A pitcher of water containing 64 ounces (2 quarts) has 1 cup (8 ounces poured from it. The ratio of cups to the pitcher is written:

$$\frac{64oz}{8oz} = \frac{64}{8} = \frac{8}{1}$$

A **ratio** is in its simplest form when the division of the units can no longer be reduced by a common factor (a number that they both have in common).

The **quotient** of two quantities that have different types of units are called **rates.** In other words, they are the result of dividing one quantity by another where the units in the quantities are not of the same kind.

Example:

A person needing to sweeten 4 quarts of tea would need to use 2 cups of sugar. The rate of sugar per quart is written:

$$\frac{2cups}{4quarts} = \frac{1cup}{2quarts}$$

Rates are in their simplest form when the two numbers on the top and bottom no longer have a common factor. The units will be used as part of the rate.

Proportions are mathematical problems (equations) that show that both sides of a rate or ratio are equal.

Examples:

$$\frac{48in}{4ft} = \frac{24in}{2ft} = \frac{12in}{1ft}$$

$$\frac{9}{15} = \frac{3}{5}$$

$$\frac{5}{9} = \frac{10}{x}$$

$$5x = 90$$

$$x = 18$$

Objective B: To Solve Application Problems

Example:

A monthly mortgage payment for a home loan is $5.63 for each $1000 borrowed. Find the mortgage payment for a $70,000 home loan using this rate.

Strategy:

Find the monthly mortgage payment by writing a **proportion**. Use M to represent the monthly mortgage payment.

Solution:

$$\frac{5.63}{1000} = \frac{M}{70000}$$

$$70000\left(\frac{5.63}{1000}\right) = 70000\left(\frac{M}{70000}\right)$$

Multiply by the common denominator of 70000

$$70\ (5.63) = M$$

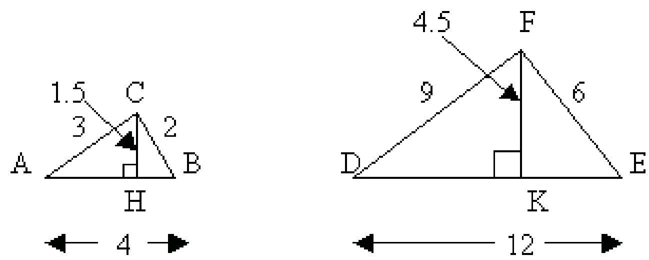
$$394.10 = M$$

Objective C: To solve problems involving similar triangles

Similar objects may have the same shape, but may not be the same size. For example, a model Corvette may have the exact same shape as an actual Corvette, but may be only a fraction of the actual size.

Similarly, two triangles might have the same shape but be different sizes.

Example:



The two triangles ABC and DEF are similar in shape, but they are obviously different in size. Although the ratios of the lengths of corresponding sides, such as the length of the lines from A to C and D to F, are the same. We can prove this by setting up an equation using the lengths of the sides shown.

$$\frac{m(\mathrm{AC})}{m(\mathrm{DF})} = \frac{3}{9} = \frac{1}{3}$$

Similar equations can be set-up and will prove the one-third ratio for all the corresponding sides and heights of the two triangles. The same thing can be said of proportions derived by forming equations involving two similar triangles.