INTRODUCTION TO EQUATIONS

- **To determine whether a given number is a solution of an equation**

An **equation** expresses the **equality of two mathematical expressions**. The expressions can be either **numerical** or **variable expressions**.

Example 1

$x + 8 = 13$

$5 + 8 = 13$ **True equation** if **"*x*"** is replaced by **"5"**

$7 + 8 = 13$ **False equation** if **"*x*"** is replaced by **"7"**

A solution of an equation is **a number** that, **when substituted for the variable**, **results in a true equation**. ***5 is a solution of the equation x+ 8 = 13***. *7 is* ***not a solution of the equation x+ 8 = 13.***

Example 2

Is **–2** a solution of $\mathbf{2x + 5 = x^2 - 3}$

$2x + 5$	=	$x^2 - 3$
$2(\mathbf{-2}) + 5$		$(\mathbf{-2})^2 - 3$
$-4 + 5$		$4 - 3$
1 = 1		

- Replace ***x*** by **-2**
- Evaluate the numerical expressions
- If the **results are equal**, **-2 is a solution of the equation**. If the **results are not equal**, **-2 is not a solution of the equation.**

- **To solve an equation of the form $x + a = b$**

To **solve an equation** means **to find a solution of the equation**. The simplest equation to solve is an equation of the form **variable = constant**, because the constant is the solution.

Addition Property of Equations

The same number can be added to each side of an equation without changing its solution. In symbols, the equation **a = b** has *the same solution as the equation* **a + c = b + c**.

Example 3

The solution of the equation ***x*** **= 5** is **5** because **5 = 5** is a **true equation**.

The solution of the equation at the right is **7** because **7 + 2 = 9** is a **true equation**

x + 2 = 9 **7 + 2 = 9**

Note that if **4** is added to each side of the equation **x + 2 = 9**, **the solution** is still **7**.

x + 2 = 9
x + 2 + 4 = 9 + 4
x + 6 = 13 **7 + 6 = 13**

If **–5** is added to each side of the equation **x + 2 = 9**, **the solution** is still **7**.

x + 2 = 9
x + 2 + (-5) = 9 + (-5)
x – 3 = 4 **7 – 3 = 43**

Example 4

Solve x – 4 = 2

x – 4 = 2 **- The goal is to rewrite the equation as variable = constant.**
x – 4 + 4 = 2 + 4 **- Add 4 to each side of the equation**.
x + 0 = 6 **- Simplify.**
x = 6 **- The equation is in the form variable = constant.**

The solution is 6.

♦ **To solve an equation of the form a*x* =b**

Multiplication Property of Equations

Each side of an equation can be multiplied by the same ***nonzero*** number without changing the solution of the equation. In symbols, **if c ≠ 0**, then **the equation a = b has the same solutions as the equation ac = bc**.

Example 5

The solution of the equation at the right **is 3** because **2 × 3 = 6** is a **true equation**.

Note that if each side of **2*x* = 6** is **multiplied by 5**, the **solution is still 3.**

If each side of **2*x* = 6** is **multiplied by –4**, the **solution is still 3**

2*x* = 6	**2 × 3 = 6**
2*x* = 6	
5(2*x*) = 5 × 6	
10*x* = 30	**10 × 3 = 30**
2*x* = 6	
(-4)(2*x*) = (-4) × 6	
-8*x* = -24	**-8 × 3 = -24**

The equation **2*x* = 6**, **10*x* = 30**, and **-8*x* = -24** are **equivalent equations**; **each equation has 3 as its solution**. These examples suggest that **multiplying each side of an equation by the same nonzero number produces equivalent equation.**

Example 6

Solve $\frac{3}{4}z = 9$

$\frac{3}{4}z = 9$	**- The goal is to rewrite the equation in the form variable = constant.**
$\frac{4}{3}\times\frac{3}{4}z = \frac{4}{3}\times 9$	**- Multiply each side of the equation by $\frac{4}{3}$.**
$1\times z = 12$	**- Simplify.**
$z = 12$	**- The equation is in the form variable = constant.**

The solution is 12.

Example 7

Solve $6x = 14$

$6x = 14$ **- The goal is to rewrite the equation in the form variable = constant.**

$\frac{6x}{6} = \frac{14}{6}$ **- Divide each side of the equation by 6.**

$x = \frac{7}{3}$ **- Simplify. The equation is in the form variable = constant.**

The solution is $\frac{7}{3}$.

When using the **Multiplication Property of Equations**, *multiply each side of the equation by the reciprocal of the coefficient when the* ***coefficient*** *is a* ***fraction****. Divide each side of the equation by the coefficient when the* ***coefficient*** *is an* ***integer*** *or* ***decimal****.*