Simplifying Variable Expressions

Definitions:

Variable terms – are terms that have an unknown value (a, b, x, …)
Constant terms – are terms that consist of only a known value (1, 4, -9, …)
Like terms – are terms with the same variable (unknown) part or constants

Examples:

- $x^2$ and $3x^2$ both of these terms have the same variable $x^2$
- $-2x$ and $7x$ both of these terms have the same variable $x$
- $9$ and $-3$ both of these terms are constants

Definition:

Distributive Property of addition - allows you to multiply (distribute) a term to a group of terms within a set of parentheses.

$$a ( b + c ) = ab + ac$$

Example:

$$5(x + 6) = 5(x) + 5(6)$$
use the distributive property to multiply each term by 5
$$= 5x + 30$$

Definition:

Associative Property of addition – allows you to rearrange the order in which three or more terms are grouped and added.

$$(a + b) + c = a + (b + c)$$

Example:

$$(x + 5) + 3 = x + (5 + 3)$$
use the associative property to group our constant terms together
$$= x + 8$$
combine like terms
Definition:

*Commutative Property of addition* – allows you to change the order in which terms are added.

\[ a + b = b + a \]

Example:

\[-5 + x^2 = x^2 + (-5) \quad \text{use the commutative property to switch the order of the terms} \]

\[ = x^2 - 5 \]

Definition:

*Addition Property of Zero* – this property just shows that adding zero to any term will leave us with the same term.

\[ a + 0 = 0 + a = a \]

Example:

\[ 8x + 6 - 8x = (8x - 8x) + 6 \quad \text{first use the associative property to group our like terms} \]

\[ = 0 + 6 \quad \text{combine like terms} \]

\[ = 6 \quad \text{use the addition property of zero} \]

Definition:

*Inverse Property of Addition* – shows that adding a term (“a”) and its additive inverse (“-a”) together will give us zero as a result.

\[ a + ( -a ) = ( -a ) + a = 0 \]

Example:

\[ 5x + (-5x) = 0 \quad \text{adding 5x with its additive inverse of -5x gives us zero} \]
Comprehensive Examples:

Simplify:

\[ 8x + 4y - 8x + y = 8x - 8x + 4y + y \]

use the commutative property to rearrange the order of the terms

\[ = (8x - 8x) + (4y + y) \]

use the associative property to group our like terms

\[ = 0 + 5y \]

combine like terms.

\[ = 5y \]

use the addition property of zero

Simplify:

\[ 4x^2 + 5x - 6x^2 - 2x + 1 = 4x^2 - 6x^2 + 5x - 2x + 1 \]

use the commutative property to rearrange the terms

\[ = (4x^2 - 6x^2) + (5x - 2x) + 1 \]

use the associative property to group like terms

\[ = -2x^2 + 3x + 1 \]

combine like terms

Definition:

**Associative property of multiplication** - allows you to rearrange the order in which three or more terms are grouped and multiplied.

\[ (a \cdot b) \cdot c = a \cdot (b \cdot c) \]

Example: \[ 5(3x) = 5 \cdot (3 \cdot x) \]

rewrite the problem indicating the multiplication

\[ = (5 \cdot 3) \cdot x \]

use the associative property to regroup the terms

\[ = 15x \]

multiply

Definition:

**Commutative property of multiplication** – allows you to change the order in which the terms are multiplied.

\[ a \cdot b = b \cdot a \]

Example: \[ (10x)(3) = 3(10x) \]

use the commutative property to move the 3 to the front

\[ = (3 \cdot 10) \cdot x \]

use the associative property to regroup the terms

\[ = 30x \]

multiply
Definition:

*Multiplication Property of One* – shows that multiplying any term by one will give us the same term.

\[ a \cdot 1 = 1 \cdot a = a \]

Example: \( 1 \cdot (5x) = (1 \cdot 5)x \) use the associative property to regroup the terms multiply. We are left with the same term (5x).

Definition:

*Inverse property of multiplication* - shows that multiplying a term (“a”) and its reciprocal, or multiplicative inverse, (“1/a”) together will give us one as a result.

\[ a \cdot 1/a = 1/a \cdot a = 1 \]

Example: \((3/2)(2x/3) = [(3/2) \cdot (2/3)]x\) use the associative property to regroup the factors
\[ = 1 \cdot x \] use the inverse property of multiplication
\[ = x \] use the multiplication property of one

**Comprehensive Examples:**

Simplify:

\[ 2(-x) = 2(-1 \cdot x) \]
\[ = [2(-1)]x \] rewrite the problem use the associative property to regroup the factors multiply
\[ = -2x \]

Simplify:

\[-1/2(4x) = [(-1/2) \cdot 4]x\] use the associative property to regroup the factors multiply
\[ = -2x \]

Simplify:

\[(16x)(2) = (2)(16x)\] use the commutative property use the associative property multiply
\[ = (2 \cdot 16)x \]
\[ = 32x \]