

Simplifying Variable Expressions

Definitions:

Variable terms – are terms that have an unknown value (a, b, x, ...)

Constant terms – are terms that consist of only a known value (1, 4, -9, ...)

Like terms – are terms with the same variable (unknown) part or constants

Examples:

x^2 and $3x^2$	both of these terms have the same variable x^2
$-2x$ and $7x$	both of these terms have the same variable x
9 and -3	both of these terms are constants

Definition:

Distributive Property of addition - allows you to multiply (distribute) a term to a group of terms within a set of parentheses.

$$a (b + c) = ab + ac$$

Example:

$$\begin{aligned} 5(x + 6) &= 5(x) + 5(6) && \text{use the distributive property to multiply each term by 5} \\ &= 5x + 30 \end{aligned}$$

Definition:

Associative Property of addition – allows you to rearrange the order in which three or more terms are grouped and added.

$$(a + b) + c = a + (b + c)$$

Example:

$$\begin{aligned} (x + 5) + 3 &= x + (5 + 3) && \text{use the associative property to group our} \\ & && \text{constant terms together} \\ &= x + 8 && \text{combine like terms} \end{aligned}$$

Definition:

Commutative Property of addition – allows you to change the order in which terms are added.

$$a + b = b + a$$

Example:

$$\begin{aligned} -5 + x^2 &= x^2 + (-5) && \text{use the commutative property to switch the order of} \\ & && \text{the terms} \\ &= x^2 - 5 \end{aligned}$$

Definition:

Addition Property of Zero – this property just shows that adding zero to any term will leave us with the same term.

$$a + 0 = 0 + a = a$$

Example:

$$\begin{aligned} 8x + 6 - 8x &= (8x - 8x) + 6 && \text{first use the associative property to group} \\ & && \text{our like terms} \\ &= 0 + 6 && \text{combine like terms} \\ &= 6 && \text{use the addition property of zero} \end{aligned}$$

Definition:

Inverse Property of Addition – shows that adding a term (“a”) and its additive inverse (“-a”) together will give us zero as a result.

$$a + (-a) = (-a) + a = 0$$

Example:

$$5x + (-5x) = 0 \quad \text{adding } 5x \text{ with its additive inverse of } -5x \text{ gives us zero}$$

Comprehensive Examples:

Simplify:

$$\begin{aligned}
 8x + 4y - 8x + y &= 8x - 8x + 4y + y && \text{use the commutative property to} \\
 & && \text{rearrange the order of the terms} \\
 &= (8x - 8x) + (4y + y) && \text{use the associative property to} \\
 & && \text{group our like terms} \\
 &= 0 + 5y && \text{combine like terms.} \\
 &= 5y && \text{use the addition property of zero}
 \end{aligned}$$

Simplify:

$$\begin{aligned}
 4x^2 + 5x - 6x^2 - 2x + 1 &= 4x^2 - 6x^2 + 5x - 2x + 1 && \text{use the commutative property} \\
 & && \text{to rearrange the terms} \\
 &= (4x^2 - 6x^2) + (5x - 2x) + 1 && \text{use the associative property to} \\
 & && \text{group like terms} \\
 &= -2x^2 + 3x + 1 && \text{combine like terms}
 \end{aligned}$$

Definition:

Associative property of multiplication - allows you to rearrange the order in which three or more terms are grouped and multiplied.

$$(a \cdot b) \cdot c = a \cdot (b \cdot c)$$

Example: $5(3x) = 5 \cdot (3 \cdot x)$ rewrite the problem indicating the multiplication

$$= (5 \cdot 3) \cdot x$$

use the associative property to regroup the terms

$$= 15x$$

multiply

Definition:

Commutative property of multiplication – allows you to change the order in which the terms are multiplied.

$$a \cdot b = b \cdot a$$

Example: $(10x)(3) = 3(10x)$ use the commutative property to move the 3 to the front

$$= (3 \cdot 10) \cdot x$$

use the associative property to regroup the terms

$$= 30x$$

multiply

Definition:

Multiplication Property of One – shows that multiplying any term by one will give us the same term.

$$a \cdot 1 = 1 \cdot a = a$$

Example: $1 \cdot (5x) = (1 \cdot 5)x$ use the associative property to regroup the terms
 $= 5x$ multiply. We are left with the same term (5x).

Definition:

Inverse property of multiplication - shows that multiplying a term (“a”) and its reciprocal, or multiplicative inverse, (“1/a”) together will give us one as a result.

$$a \cdot 1/a = 1/a \cdot a = 1$$

Example: $(3/2)(2x/3) = [(3/2) \cdot (2/3)]x$ use the associative property to regroup the factors
 $= 1 \cdot x$ use the inverse property of multiplication
 $= x$ use the multiplication property of one

Comprehensive Examples:

Simplify:

$2(-x) = 2(-1 \cdot x)$ rewrite the problem
 $= [2(-1)]x$ use the associative property to regroup the factors
 $= -2x$ multiply

Simplify:

$-1/2(4x) = [(-1/2) \cdot 4]x$ use the associative property to regroup the factors
 $= -2x$ multiply

Simplify:

$(16x)(2) = (2)(16x)$ use the commutative property
 $= (2 \cdot 16)x$ use the associative property
 $= 32x$ multiply