Factoring a Monomial from a Polynomial

Greatest Common Factor (most popular as **GCF**)

We need to recall what the factors of a number are. The **factor of a number is any number that divides into another number evenly.**

24 can be divided by **1, 2, 3, 4, 6**, 8, **12**, and 24.
So 1, 2, 3, 6, 8, 12, and 24 are factors of 24

36 can be divided by **1, 2, 3, 4**, 6, 9, **12**, 18, and 36.
So 1, 2, 3, 4, 6, 9, 12, and 36 are factors of 36.

Did you realize that 1,2, 3, 4, 6, and 12 are common factors of 24 and 36?
So the **Greatest Common Factor** of 24 and 36 is **12.**

To find the **GCF** of 24 and 36 using prime factorization:

1. Write the prime factorization of each number

```
      24
      /\
   2      12
          /\
       2      6
              /\
           2      3
```

$24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3$

```
      36
      /\
   2      18
          /\
       2      9
              /\
           3      3
```

$36 = 2 \times 2 \times 3 \times 3 = 2^2 \times 3^2$

2. Select the **LOWEST** power of each prime factor.

$24 = 2^3 \times \mathbf{3}$
$36 = \mathbf{2^2} \times 3^2$

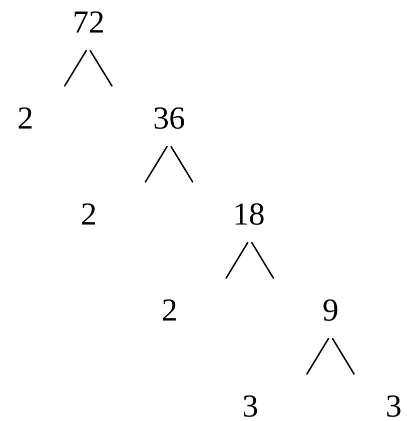
3. The **GCF** is the product of the selected factors (in bold):

   $2^2 \times 3 = 12$

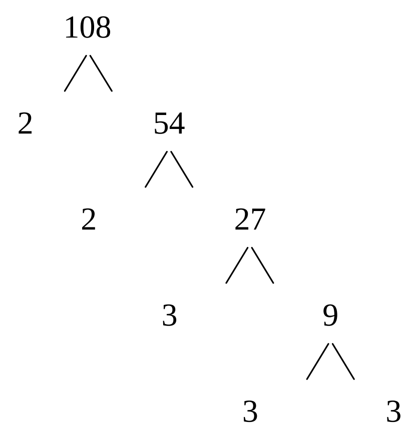
Example 1: Find the GCF for 72 and 108.

Solution:

Step 1: Prime factor each of the numbers



$72 = 2 \times 2 \times 2 \times 3 \times 3 = 2^3 \times 3^2$



$108 = 2 \times 2 \times 3 \times 3 \times 3 = 2^2 \times 3^3$

Step 2: Select the lowest power of each prime factor

$72 = 2^3 \times \mathbf{3^2}$
$108 = \mathbf{2^2} \times 3^3$

Step 3: The GCF is the product of the selected factors

GCF: $2^2 \times 3^2 = 36$

The same process is performed when dealing with variables or groups of terms. You will look at the variables and select the lowest power of the common variables.

Example 2: Find the GCF for $12x^2y^2$, $9xy^4$, and $6x^2y^3$.

Solution:

Step 1: Write the prime factorization for each term

$12x^2y^2 = 2^2 \times 3 \times x^2 \times y^2$
$9xy^4 = 3^2 \times x \times y^4$
$6x^2y^3 = 2 \times 3 \times x^2 \times y^3$

Step 2: Select the lowest power of each factor

$12x^2y^2 = 2^2 \times 3 \times x^2 \times \mathbf{y^2}$
$9xy^4 = 3^2 \times \mathbf{x} \times y^4$
$6x^2y^3 = \mathbf{2} \times \mathbf{3} \times x^2 \times y^3$

Step 3: The GCF is the product of the selected factors

GCF: $2 \times 3 \times x \times y^2 = 6xy^2$

Example 3: Find the GCF for $6(x - 3)$ and $3x(x - 3)$.

Solution:

Step 1: Write the prime factorization for each term

$6(x - 3) = 2 \times 3 \times (x - 3)$
$3x(x - 3) = 3 \times x \times (x - 3)$

Step 2: Select the lowest power of each factor

$6(x - 3) = 2 \times 3 \times \mathbf{(x - 3)}$
$3x(x - 3) = \mathbf{3} \times x \times (x - 3)$

Step 3: The GCF is the product of the selected factors

GCF: $3 \times (x - 3) = 3(x - 3)$

We can now use the process of finding the GCF and the distributive property of multiplication to factor common terms out of polynomials. The distributive property of multiplication tells us that $a(b + c) = ab + ac$. We are actually using the reverse of this property when we "factor out" our GCF from the polynomial.

Example 4: Factor $55y^3 – 20y^2$

Solution:

Step 1: Prime factor our terms in the polynomial

$55y^3 = 5 \times 11 \times y^3$
$20y^2 = 2^2 \times 5 \times y^2$

Step 2: Select the lowest power of each factor to determine our GCF

$55y^3 = \mathbf{5} \times 11 \times y^3$
$20y^2 = 2^2 \times 5 \times \mathbf{y^2}$

GCF: $5 \times y^2 = 5y^2$

Step 3: Rewrite each term in the polynomial as the product of the GCF and some other factor.

$55y^3 = 5 \times 11 \times y^3 = 5y^2 \times 11y$
$20y^2 = 2^2 \times 5 \times y^2 = 5y^2 \times 4$

$55y^3 – 20y^2 = 5y^2 \times 11y – 5y^2 \times 4$

Step 4: Now use the reverse of the distributive property to factor out our GCF

$55y^3 – 20y^2 = 5y^2 \times 11y – 5y^2 \times 4$

The GCF is placed outside the parenthesis and the remaining factors are placed inside.

$55y^3 – 20y^2 = 5y^2 (11y – 4)$
$= 5y^2 (11y – 4)$