Factoring Trinomials of the Form $ax^2 + bx + c, a \neq 1$

When trinomials factor, the resulting terms are binomials. To help establish a procedure for solving these types of equations look at the following patterns.

PATTERN I. **$AX^2 + BX + C$.**

In this pattern A is positive and both of the operators are addition. This will result in two monomials, both with operators of addition (+), (a + b) (c + d).

PATTERN II. **$AX^2 - BX + C$.**

In this pattern A is positive, the operator before the B term is subtraction (-) and the operator before the C term is addition (+). This will result in two monomials, both with operators of subtraction (-), (a – b) (c – d).

PATTERN III. **$AX^2 \pm BX - C$.**

There are two patterns shown above, either of which will give the same result. The conditions needed to fit this pattern are that A is positive and that the operator before the C term is subtraction (-). The operator before the B term may be either a subtraction or addition. This will result in two monomials, one with an operation of addition and one of subtraction, (a + b) (c – d).

Example 1. Factor $3x^2 + 14x + 8$.

This polynomial fits pattern 1, so both monomial factors will contain only addition operators. This will be important to remember as we solve the equation using the steps below.

Step 1. Multiply the A term (3) and the constant C (8).
8 * 3 = 24

Step 2. Factor the product from step 1 (24 in this example), and arrange them into two columns, one decreasing and one increasing.

24	**1**
12	**2**
8	**3**
6	**4**
4	**6**

(when the pattern repeats you may stop)

Step 3. Since we have determined that the problem fits pattern 1, all of the values in both columns are positive. Your next step is to add across, looking for a sum that is equal to B (14 in this case).

24	**1**	**= 25**
12	**2**	**= 14**
8	**3**	**= 11**
6	**4**	**= 10**

(This is the set we need)

NOTE : If none of the sums are equal to B, either the equation is prime or has fractional factors.

Step 4. Rewrite the original equation substituting the set we selected from step 3 for the B term.

$$3x^2 + 14x + 8 = 3x^2 + 12x + 2x + 8$$

Step 5. Group the first and last two term.

$$3x^2 + 12x + 2x + 8 = (3x^2 + 12x) + (2x + 8)$$

This process is known as *"factoring by grouping"*.

Note: If the operator between the groups is a subtraction, the operator in the second grouping *must* be changed, i.e. addition becomes subtraction or subtraction becomes addition.

Step 6. Factor out whatever is common within each term.

$$(3x^2 + 12x) + (2x + 8)$$

$$3x(x + 4) + 2(x + 4)$$

Note: If the grouping are not alike, there is no real solution.

Step 7. Factor the common grouping and group the remaining factors.

$$3x(x + 4) + 2(x + 4)$$

$$(x + 4)(3x + 2)$$ **[These are the factors of the problem]**

Example 2. Factor $6x^2 – 19x + 10$.

This problem fits pattern 2. The procedure to solve it is identical to those in the first example except that both columns of numbers are negative since both of the operators in the answer will be subtraction.

Step 1. **6 * 10 = 60**

Step 2.

60	**1**
30	**2**
20	**3**
15	**4**
12	**5**
10	**6**

Step 3.

60	**1**	**=**	**- 61**
30	**2**	**=**	**- 32**
20	**3**	**=**	**- 23**
15	**4**	**=**	**- 19**
12	**5**	**=**	**- 17**
10	**6**	**=**	**- 16**

(These are the factors)

Step 4. $6x^2 - 19x + 10 = 6x^2 - 15x - 4x + 10$

Step 5. $6x^2 - 15x - 4x + 10 = (6x^2 - 15x) - (4x - 10)$

Note: The operator in the second grouping has changed from addition to subtraction because the operator between the groups is subtraction.

Step 6. $(6x^2 - 15x) - (4x - 10)$

$3x (2x - 5) - 2 (2x - 5)$

Step 7. $(2x - 5) (3x - 2)$ **[These are the factors]**

Example 3. Factor $6x^2 + 7x - 3$.

This problem fits the 3rd pattern. A clear indication that this is the pattern is that A is positive *and* that the operator before the constant (3) is a subtraction. The same steps are followed that were used in examples one and two with only one change. This pattern states that one of the monomial factors contains an addition operator while the other has a subtraction. This means that one of the factor columns is positive while the other is negative. To determine which column is which, look at the operator before the B term (7). If it is an addition, like in this example, the descending column becomes positive and the ascending column negative. Should the operator have been subtraction then the descending column would have been negative and the ascending column positive.

Step 1. **6 * 3 = 18**

Step 2.

18	**1**
9	**2**
6	**3**

Step 3.

18	**- 1**	**=**	**17**	
9	**- 2**	**=**	**7**	**(these are the factors)**
6	**- 3**	**=**	**3**	

Step 4. The factors selected above include one that is positive and one that is negative. When they are substituted in to the equation for B it is desirable to write the negative before the positive factor so that you will not be forced to change the operator in the second factor as you were in the last example.

$$6x^2 + 7x - 3 = 6x^2 - 2x + 9x - 3$$

Step 5. $6x^2 - 2x + 9x - 3 = (6x^2 - 2x) + (9x - 3)$

Step 6. $(6x^2 - 2x) + (9x - 3)$

$2x (3x - 1) + 3 (3x - 1)$

Step 7. $(3x - 1) (2x + 3)$

[These are the factors of the problem]

Example 4. **Factor $6x^2 - 7x - 3$.**

This problem also fits pattern 3. The difference in this case is that, since B is proceeded by a subtraction operator, the descending column is negative and the ascending column is positive. All steps are done in the same manner.

Step 1. **6 * 3 = 18**

Step 2.

18	**1**
9	**2**
6	**3**

Step 3.

-18	**1**	**=**	**-17**
-9	**2**	**=**	**-7** **(these are the terms)**
-6	**3**	**=**	**-3**

Step 4. $6x^2 - 7x - 3 = 6x^2 - 9x + 2x - 3$

Step 5. $6x^2 - 9x + 2x - 3 = (6x^2 - 9x) + (2x - 3)$

Step 6. $(6x^2 - 9x) + (2x - 3)$

$3x(2x - 3) + \mathit{1}(2x - 3)$

Note: If there are no obvious factors, factor 1 as shown above.

Step 7. $(2x - 3)(3x + 1)$