Factoring by Grouping

In the previous section, we learned how to use the GCF to factor polynomials with two or three terms. Now we will look at the situation where the given polynomial has four terms where there may or may not be a GCF between all of the terms. In order to factor four term polynomials we will use a process called "factoring by grouping." Factoring by grouping is a process of grouping the terms together in pairs of two terms so that each pair of terms has a common factor that we can factor out.

Steps in factoring by grouping:

1. Determine if there is a GCF common to all four terms. If there is one then begin by factoring out this GCF.
2. Arrange the four terms so that the first two terms and the last two terms have common factors.
3. If the coefficient of the third term is negative, factor out a negative coefficient from the last two terms.
4. Use the reverse of the distributive property to factor each group of two terms.
5. Now factor the GCF from the result of step 4 as done in the previous section.

Example 1: Factor $x^2 - 3x + 4x - 12$ by grouping.

Solution:

Step 1: Factor out the GCF common to all four terms (if there is one).

$x^2 = x^2$
$3x = 3 \times x$
$4x = 2^2 \times x$
$12 = 2^2 \times 3$

GCF: none

Step 2: Arrange the terms so that the first two and last two have a common factor.

The first two terms already have x as a common factor

$x^2 = x \times \mathbf{x}$
$3x = 3 \times \mathbf{x}$

The last two terms have 2^2 (or 4) as a common factor

$4x = \mathbf{2^2} \times x$
$12 = \mathbf{2^2} \times 3$

So we do not need to rearrange the order of the terms.

Example 1 (Continued):

Step 3: If the coefficient of the third term is negative, factor out a negative coefficient from the last two terms.

The coefficient of the third term in the polynomial (4x) is positive so we do not need to factor out a negative coefficient.

Step 4: Use the reverse of the distributive property to factor each group of two terms.

$$\begin{aligned} x^2 - 3x + 4x - 12 &= (x^2 - 3x) + (4x - 12) \\ &= (x \times x - 3 \times x) + (4 \times x - 3 \times 4) \\ &= x(x - 3) + 4(x - 3) \end{aligned}$$

Step 5: Now factor the GCF from the result of step 4.

$x(x - 3) = x \times \mathbf{(x - 3)}$
$4(x - 3) = 4 \times \mathbf{(x - 3)}$

GCF: $(x - 3)$

$$\begin{aligned} x^2 - 3x + 4x - 12 &= (x^2 - 3x) + (4x - 12) \\ &= (x \times x - 3 \times x) + (4 \times x - 3 \times 4) \\ &= x(x - 3) + 4(x - 3) \\ &= (x - 3)(x + 4) \end{aligned}$$

$x^2 - 3x + 4x - 12 = (x - 3)(x + 4)$

Example 2: Factor $2x^2 + 3xy - 8xy - 12y^2$ by grouping.

Solution:

Step 1: Factor out the GCF common to all four terms (if there is one).

$2x^2 = 2 \times x^2$
$3xy = 3 \times x \times y$
$8xy = 2^3 \times x \times y$
$12y^2 = 2^2 \times 3 \times y^2$

GCF: none

Example 2 (Continued):

Step 2: Arrange the terms so that the first two and last two have a common factor.

The first two terms already have x as a common factor

$2x^2 = 2 \times x^2 = 2x \times \mathbf{x}$
$3xy = 3 \times \mathbf{x} \times y = 3y \times \mathbf{x}$

The last two terms have $2^2 \times y$ (or 4y) as a common factor

$8xy = 2^3 \times x \times \mathbf{y} = 2x \times \mathbf{2^2y}$
$12y^2 = \mathbf{2^2} \times 3 \times y^2 = 3y \times \mathbf{2^2y}$

So we do not need to rearrange the order of the terms.

Step 3: If the coefficient of the third term is negative, factor out a negative coefficient from the last two terms.

$$\begin{aligned}2x^2 + 3xy - 8xy - 12y^2 &= 2x^2 + 3xy + (-1)8xy + (-1)12y^2\\ &= (2x^2 + 3xy) - 1(8xy + 12y^2)\\ &= (2x^2 + 3xy) - (8xy + 12y^2)\end{aligned}$$

Step 4: Use the reverse of the distributive property to factor each group of two terms.

$$\begin{aligned}2x^2 + 3xy - 8xy - 12y^2 &= 2x^2 + 3xy + (-1)8xy + (-1)12y^2\\ &= (2x^2 + 3xy) - 1(8xy + 12y^2)\\ &= (2x \times x + 3y \times x) - (2x \times 4y + 3y \times 4y)\\ &= x(2x + 3y) - 4y(2x + 3y)\end{aligned}$$

Step 5: Now factor the GCF from the result of step 4.

$x(2x + 3y) = x \times \mathbf{(2x + 3y)}$
$4y(2x + 3y) = 4y \times \mathbf{(2x + 3y)}$

GCF: $(2x + 3y)$

$$\begin{aligned}2x^2 + 3xy - 8xy - 12y^2 &= 2x^2 + 3xy + (-1)8xy + (-1)12y^2\\ &= (2x^2 + 3xy) - 1(8xy + 12y^2)\\ &= (2x \times x + 3y \times x) - (2x \times 4y + 3y \times 4y)\\ &= x(2x + 3y) - 4y(2x + 3y)\\ &= (2x + 3y)(x - 4y)\end{aligned}$$

$$2x^2 + 3xy - 8xy - 12y^2 = (2x + 3y)(x - 4y)$$