Solving Quadratic Equations Using Factoring

Once polynomials have been factored, we may begin to solve for their critical points before they are graphed. These points would include such things as x and y intercepts and flex points. To accomplish this, the equation is first set equal to zero and then each of the factors are solved for the appropriate variable.

Example 1. Solve $x^2 + 5x + 6$ for $x$.

**Step 1.** Set the equation equal to zero.
$$x^2 + 5x + 6 = 0$$

**Step 2.** Factor the polynomial (see 4.6)
$$(x + 2)(x + 3) = 0$$

**Step 3.** Set each term equal to zero.
$$(x + 2) = 0 \text{ or } (x + 3) = 0$$

**Step 4.** Solve for the indicated variable.
$$x + 2 = 0 \text{ or } x + 3 = 0$$
$$x = -2 \text{ or } x = -3$$

The solution set for this problem is $\{ -2, -3 \}$

Example 2. Solve $x^2 - 5x + 6$ for $x$.

**Step 1.**
$$x^2 - 5x + 6 = 0$$

**Step 2.**
$$(x - 2)(x - 3) = 0$$

**Step 3.**
$$(x - 2) = 0 \text{ or } (x - 3) = 0$$

**Step 4.**
$$x - 2 = 0 \text{ or } x - 3 = 0$$
$$x = 2 \text{ or } x = 3$$
$$\{2, 3\}$$
Example 3. Solve $x^2 + 6x + 9$ for $x$.

**Step 1.** $x^2 + 6x + 9 = 0$

**Step 2.** $(x + 3)(x + 3) = 0$

**Step 3.** $x + 3 = 0$ or $x + 3 = 0$

**Step 4.** $x = -3$ or $x = -3$

$\{ -3 \}$

**Note:** The solutions for this equation are both $-3$. When the same solution is found a number of times it is said to have a zero of multiplicity however many times that value is repeated. In this example it is said that the equation has “a zero of $-3$ of multiplicity 2”. 