

SOLVING QUADRATIC EQUATIONS BY COMPLETING THE SQUARE

Using the square root property it is possible to solve any quadratic equation written in the form $(x + b)^2 = c$. The key to setting these problems into the correct form is to recognize that $(x + b)^2$ is a perfect square trinomial. To turn the equation given into one that can be solved using the square root property, the following must be done: Given : $ax^2 + bx + c = 0$

- 1.) If $a \neq 1$ divide both sides by a .
- 2.) Rewrite the equation so that both terms containing variables are on one side of the equation and the constant is on the other.
- 3.) Take half of the coefficient of x and square it.
- 4.) Add the square to both sides.
- 5.) One side should now be a perfect square trinomial. Write it as the square of a binomial.
- 6.) Use the square root property to complete the solution.

Example 1. Solve $2a^2 - 4a - 5 = 0$ by completing the square.

$$\text{Step 1. } a^2 - 2a - \frac{5}{2} = 0$$

$$\text{Step 2. } a^2 - 2a = \frac{5}{2}$$

$$\text{Step 3. } \left(\frac{1}{2}\right)\left(\frac{-2}{1}\right) = -1$$

$$(-1)^2 = 1$$

$$\text{Step 4. } a^2 - 2a + 1 = \frac{5}{2} + 1$$

$$\text{Step 5. } (a - 1)^2 = \frac{7}{2}$$

$$\text{Step 6. } \sqrt{(a-1)^2} = \pm \sqrt{\frac{7}{2} \left(\sqrt{\frac{2}{2}} \right)} = \pm \frac{\sqrt{14}}{2}$$

$$(a-1) = \pm \frac{\sqrt{14}}{2}$$

$$a = 1 \pm \frac{\sqrt{14}}{2}$$

$$a = \frac{2}{2} + \frac{\sqrt{14}}{2} \quad \text{or} \quad a = \frac{2}{2} - \frac{\sqrt{14}}{2}$$

$$a = \frac{2 + \sqrt{14}}{2} \quad \text{or} \quad a = \frac{2 - \sqrt{14}}{2}$$

Example 2. Solve $9a^2 - 24a = -13$ by completing the square.

$$\text{Step 1. } a^2 - \frac{24}{9}a = -\frac{13}{9}$$

$$\text{Step 2. } a^2 - \frac{8}{3}a = -\frac{13}{9}$$

$$\text{Step 3. } \left(\frac{1}{2} \right) \left(\frac{-8}{3} \right) = -\frac{8}{6} = -\frac{4}{3}$$

$$\left(\frac{-4}{3} \right)^2 = \frac{16}{9}$$

$$\text{Step 4. } a^2 - \frac{8}{3}a + \frac{16}{9} = -\frac{13}{9} + \frac{16}{9}$$

$$a^2 - \frac{8}{3}a + \frac{16}{9} = \frac{3}{9}$$

$$a^2 - \frac{8}{3}a + \frac{16}{9} = \frac{1}{3}$$

$$\text{Step 5. } \left(a - \frac{4}{3}\right)^2 = \frac{1}{3}$$

$$\text{Step 6. } \sqrt{\left(a - \frac{4}{3}\right)^2} = \pm \sqrt{\frac{1}{3}\left(\frac{\sqrt{3}}{\sqrt{3}}\right)} = \pm \frac{\sqrt{3}}{3}$$

$$a - \frac{4}{3} = \pm \frac{\sqrt{3}}{3}$$

$$a = \frac{4}{3} \pm \frac{\sqrt{3}}{3}$$

$$a = \frac{4 + \sqrt{3}}{3} \quad \text{or} \quad a = \frac{4 - \sqrt{3}}{3}$$

Example 3. Solve $9x^2 - 30x + 29$ by completing the square.

$$\text{Step 1. } x^2 - \frac{30}{9}x + \frac{29}{9} = 0$$

$$\text{Step 2. } x^2 - \frac{10}{3}x = -\frac{29}{9}$$

$$\text{Step 3. } \left(\frac{1}{2}\right)\left(-\frac{10}{3}\right) = -\frac{10}{6} = -\frac{5}{3}$$

$$\left(-\frac{5}{3}\right)^2 = \frac{25}{9}$$

$$\text{Step 4. } x^2 - \frac{10}{3}x + \frac{25}{9} = \frac{25}{9} - \frac{29}{9}$$

$$x^2 - \frac{10}{3}x + \frac{25}{9} = -\frac{4}{9}$$

$$\text{Step 5. } \left(x - \frac{5}{3}\right)^2 = -\frac{4}{9}$$

$$\text{Step 6. } \sqrt{\left(x - \frac{5}{3}\right)^2} = \pm\sqrt{\frac{-4}{9}}$$

$$x - \frac{5}{3} = \pm\frac{2i}{3}$$

$$x = \frac{5}{3} \pm \frac{2i}{3}$$

$$x = \frac{5 + 2i}{3} \quad \text{or} \quad x = \frac{5 - 2i}{3}$$