Addition and Subtraction of Rational Expressions

When looking to add or subtract rational expressions you must make sure that the denominators are the same before attempting to add or subtract the numerators. This section will review over problems where the denominators are not alike and a LCD must be obtained. In some rational expressions the denominators of the fractions will be polynomials that must be factored before determining the LCD.

Steps for adding/subtracting rational expressions with different denominators:

1. Factor any polynomial denominators
2. Determine the LCD by comparing the factors in each denominator
3. Multiply each fraction by the missing factors in the LCD as an equivalent fraction of 1
4. Distribute the missing factors to the numerators
5. Add or subtract the numerators over the LCD
6. Simplify the resulting fraction

Example 1: Add or subtract the rational expressions.

$$\frac{x-3}{x}+\frac{7}{x+2}$$

Solution:

Step 1: Factor any polynomial denominators

In this problem, the denominators are already factored.

Step 2: Determine the LCD

Denominators: x and x + 2
LCD: x(x + 2)

Step 3: Multiply each fraction by the missing factors in the LCD as an equivalent fraction of 1.

The denominator of the first fraction is missing the "x + 2" factor so we would multiply it by (x + 2)/(x + 2).

The denominator of the second fraction is missing the "x" factor so we would multiply it by x/x.

$$\frac{x-3}{x}+\frac{7}{x+2}=\frac{x-3}{x}\times\frac{x+2}{x+2}+\frac{7}{x+2}\times\frac{x}{x}$$

Example 1 (Continued):

Step 4: Distribute the missing factors to the numerators

$$\begin{aligned}\frac{x-3}{x}+\frac{7}{x+2} &= \frac{x-3}{x}\times\frac{x+2}{x+2}+\frac{7}{x+2}\times\frac{x}{x}\\ &= \frac{(x-3)(x+2)}{x(x+2)}+\frac{7(x)}{(x+2)(x)}\\ &= \frac{x^2+2x-3x-6}{x(x+2)}+\frac{7x}{(x+2)(x)}\\ &= \frac{x^2-x-6}{x(x+2)}+\frac{7x}{x(x+2)}\end{aligned}$$

Step 5: Add or subtract the numerators over the LCD

$$\begin{aligned}\frac{x-3}{x}+\frac{7}{x+2} &= \frac{x^2-x-6}{x(x+2)}+\frac{7x}{x(x+2)}\\ &= \frac{(x^2-x-6)+(7x)}{x(x+2)}\\ &= \frac{x^2-x-6+7x}{x(x+2)}\\ &= \frac{x^2+6x-6}{x(x+2)}\end{aligned}$$

Step 6: Simplify the resulting fraction

The numerator will not factor so this fraction is simplified as far as it can go.

$$\frac{x-3}{x}+\frac{7}{x+2}=\frac{x^2+6x-6}{x(x+2)}$$

Example 2: Add or subtract the rational expressions.

$$\frac{x-2}{x^2-x-6}-\frac{x+3}{x^2-8x+15}$$

Solution

Step 1: Factor any polynomial denominators

$x^2 - x - 6 = (x + 2)(x - 3)$

Factors of -6	Sum of factors
1 * -6 = -6	1 + -6 = -5
2 * -3 = -6	2 + -3 = -1
-1 * 6 = -6	-1 + 6 = 5
-2 * 3 = -6	-2 + 3 = 1

$x^2 - 8x + 15 = (x - 3)(x - 5)$

Factors of 15	Sum of factors
1 * 15 = 15	1 + 15 = 16
3 * 5 = 15	3 + 5 = 8
-1 * -15 = 15	-1 + -15 = -16
-3 * -5 = 15	-3 + -5 = -8

$$\frac{x-2}{x^2-x-6}-\frac{x+3}{x^2-8x+15}=\frac{x-2}{(x+2)(x-3)}-\frac{x+3}{(x-3)(x-5)}$$

Step 2: Determine the LCD

Denominators: (x + 2)(x – 3) and (x – 3)(x – 5)
LCD: (x + 2)(x – 3)(x – 5)

Step 3: Multiply each fraction by the missing factors in the LCD as an equivalent fraction of 1.

The denominator of the first fraction is missing the "x – 5" factor so we would multiply it by (x – 5)/(x – 5).

The denominator of the second fraction is missing the "x + 2" factor so we would multiply it by (x + 2)/(x + 2).

$$\frac{x-2}{x^2-x-6}-\frac{x+3}{x^2-8x+15}=\frac{x-2}{(x+2)(x-3)}-\frac{x+3}{(x-3)(x-5)}$$
$$=\frac{x-2}{(x+2)(x-3)}\times\frac{(x-5)}{(x-5)}-\frac{x+3}{(x-3)(x-5)}\times\frac{(x+2)}{(x+2)}$$

Example 2 (Continued):

Step 4: Distribute the missing factors to the numerators

$$\frac{x-2}{x^2-x-6}-\frac{x+3}{x^2-8x+15}=\frac{x-2}{(x+2)(x-3)}\times\frac{(x-5)}{(x-5)}-\frac{x+3}{(x-3)(x-5)}\times\frac{(x+2)}{(x+2)}$$
$$=\frac{(x-2)(x-5)}{(x+2)(x-3)(x-5)}-\frac{(x+3)(x+2)}{(x-3)(x-5)(x+2)}$$
$$=\frac{x^2-5x-2x+10}{(x+2)(x-3)(x-5)}-\frac{x^2+2x+3x+6}{(x-3)(x-5)(x+2)}$$
$$=\frac{x^2-7x+10}{(x+2)(x-3)(x-5)}-\frac{x^2+5x+6}{(x-3)(x-5)(x+2)}$$

Step 5: Add or subtract the numerators over the LCD

$$\frac{x-2}{x^2-x-6}-\frac{x+3}{x^2-8x+15}=\frac{x^2-7x+10}{(x+2)(x-3)(x-5)}-\frac{x^2+5x+6}{(x-3)(x-5)(x+2)}$$
$$=\frac{(x^2-7x+10)-(x^2+5x+6)}{(x+2)(x-3)(x-5)}$$
$$=\frac{x^2-7x+10-x^2-5x-6}{(x+2)(x-3)(x-5)}$$
$$=\frac{-12x+4}{(x+2)(x-3)(x-5)}$$

Step 6: Simplify the resulting fraction

$$\frac{x-2}{x^2-x-6}-\frac{x+3}{x^2-8x+15}=\frac{-12x+4}{(x+2)(x-3)(x-5)}$$
$$=\frac{-4(3x-1)}{(x+2)(x-3)(x-5)}$$

We can factor a "– 4" out of the numerator. There are no common factors between the numerator and denominator so this is as far as the fraction can be

reduced.