COMPLEX FRACTIONS

A complex fraction is one in which the numerator and/or denominator of a fraction is itself a fraction. Some examples of this are:

$$\frac{\frac{3}{5}}{\frac{2}{4}}, \quad \frac{\frac{1}{3}+\frac{2}{5}}{\frac{1}{4}+\frac{1}{2}}, \quad \frac{\frac{1}{a}+\frac{2}{b}}{c}, \quad \frac{3}{\frac{1}{c}+\frac{2}{d}-\frac{e}{4}}$$

One of the final steps in solving this type of probe is to invert the denominator and then multiply it by the numerator, after which the product is reduced. To allow this to happen the objective of these types of problems is to manipulate the problem so that the numerator and denominator are single rational terms. The following examples will show how this may be done.

Example 1. **Simplify** $\frac{\frac{3}{a}}{\frac{5}{ab}}$ **.**

Step 1. **Since the numerator and denominator are already single fractional terms, the denominator is inverted and then factored.**

$$\left(\frac{3}{a}\right)\left(\frac{ab}{5}\right)$$

Step 2. **Group the like term to reduce.**

$$\left(\frac{3}{1}\right) * \left(\frac{a}{a}\right) * \left(\frac{b}{1}\right) * \left(\frac{1}{5}\right)$$

Step 3. Simplify.

$$\left(\frac{3}{1}\right) * (1) * \left(\frac{b}{1}\right) * \left(\frac{1}{5}\right)$$

Step 4. Multiply.

$\frac{3b}{5}$ **This is the solution in lowest terms.**

In this example find the LCD of the numerator, then the denominator, separately, then simplify the problem.

Example 2. Simplify $\dfrac{\frac{2}{5}+\frac{5}{6}}{\frac{2}{3}+\frac{3}{10}}$

Step 1. Find the LCD of the terms in the numerator and denominator and multiply by forms of 1 to convert the terms to the LCD.

Numerator LCD = 5 * 2 * 3 = 30 Denominator LCD = 3 * 2 * 5 = 30

$$\frac{\left(\frac{2}{5}\right)\left(\frac{6}{6}\right)+\left(\frac{5}{6}\right)\left(\frac{5}{5}\right)}{\left(\frac{2}{3}\right)\left(\frac{10}{10}\right)+\left(\frac{3}{10}\right)\left(\frac{3}{3}\right)}$$

Step 2. Simplify.

$$\frac{\frac{12}{30}+\frac{25}{30}}{\frac{20}{30}+\frac{9}{30}} = \frac{\frac{37}{30}}{\frac{29}{30}}$$

Step 3. Invert and simplify.

$$\frac{\frac{37}{30}}{\frac{29}{30}} = \left(\frac{37}{30}\right)\left(\frac{30}{29}\right) = \left(\frac{30}{30}\right)\left(\frac{37}{29}\right)$$

$$= \mathbf{1} * \left(\frac{37}{29}\right)$$

$$= \left(\frac{37}{29}\right)$$ **This is the solution in lowest terms.**

In the next example the LCD for all of the terms are first found and then multiplied to each term. This results in the loss of all fractions in the problem. The remaining terms are then factored to see if the solution may be further reduced.

Example 3. Simplify $\dfrac{\frac{x}{3}+\frac{4}{y}}{\frac{3}{5x^2}+\frac{5}{y^3}}$ **.**

Step 1. Find the LCD of the problem's terms.

The LCD = 3 * y * y * y* 5 * x * x = 15 x^2 y^3 .

Step 2. Multiply the numerator and the denominator by the LCD.

$$\left(\frac{15x^2y^3}{15x^2y^3}\right)\ \frac{\frac{x}{3}+\frac{4}{y}}{\frac{3}{5x^2}+\frac{5}{y^3}}$$

Step 3. Reduce.

$$\frac{\left(5x^2y^3\right)(x)+\left(15x^2y^2\right)(4)}{\left(3y^3\right)(3)+\left(15x^2\right)(5)}$$

Step 4. Simplify.

$$\frac{15x^3y^3 + 60x^2y^2}{9y^3 + 75x^2}$$

Step 5. Factor.

$$\frac{(15x^2y^2)(xy+4)}{(3)(y^3+25x^2)}$$

Step 6. Reduce.

$$\frac{5x^2y^2(xy+4)}{y^3+25x^2}$$

The last example explores a method to use to solve a problem that has a fraction as one of its terms in a "deep" layer. The method in question solves the equation from the innermost fractions to the outer layer, by finding the LCD's of that layer.

Example 4. Simplify $2 - \dfrac{x}{2 - \frac{2}{x}}$.

Step 1. Group and expand each fractional term.

$$\left(\frac{2}{1}\right) - \left(\frac{\frac{x}{1}}{\frac{2}{1} - \frac{2}{x}}\right)$$

Step 2. Find the LCD of the innermost fraction.

$$\left(\frac{2}{1}\right) - \left(\frac{\frac{x}{1}}{\left(\frac{2}{1}\right)\left(\frac{x}{x}\right) - \frac{2}{x}}\right)$$

Step 3. Simplify.

$$\left(\frac{2}{1}\right)-\left(\frac{\frac{x}{1}}{\frac{2x}{x}-\frac{2}{x}}\right)$$

Step 4. Continue.

$$\left(\frac{2}{1}\right)-\left(\frac{\frac{x}{1}}{\frac{2x-2}{x}}\right)$$

Step 5. Invert and multiply.

$$\left(\frac{2}{1}\right)-\left(\frac{x}{1}\right)\left(\frac{x}{2x-2}\right) = \left(\frac{2}{1}\right)-\left(\frac{x^2}{2x-2}\right)$$

Step 6. Find the LCD of both terms.

$$\left(\frac{2}{1}\right)\left(\frac{2x-2}{2x-2}\right)-\left(\frac{x^2}{2x-2}\right)$$

Step 7. Multiply and solve.

$$\frac{4x-4}{2x-2}-\frac{x^2}{2x-2} = \frac{4x-4-x^2}{2x-2} = \frac{-\left(x^2-4x+4\right)}{2x-2}$$

This is the final solution because there are no common factors left in the problem.