MULTIPLYING AND DIVIDING RATIONAL EXPRESSIONS

Recall that the definition of multiplication of rational numbers in common fraction form states that:

$\frac{a}{b} * \frac{c}{d} = \frac{a * c}{b * d} = \frac{ac}{bd}$ **, where a, b, c and d are integers and b and d $\neq$ 0 .**

This definition is similar to the fundamental principle of fractions that was used to simplify rational expressions.

Example 1. **Multiply and simplify** $\frac{2}{3} * \frac{5}{7}$ **.**

Step 1. Determine the sign of the solution.

Since both terms are positive the solution will be positive.

Step 2. Factor each fraction.

$$\frac{2 * 5}{3 * 7}$$

All of the factors of the numerator and denominator are already prime.

Step 3. Combine like terms and reduce to forms of 1.

No factors are alike.

Step 4. Multiply all numerators together and all denominators together. Their product will be the solution.

$\frac{2 * 5}{3 * 7} = \frac{10}{21}$ **This is the solution.**

Example 2. Multiply and simplify $\frac{2}{5} * \frac{-10}{12}$ **.**

Step 1. Determine the sign of the solution.

Since one term is positive and one is negative the solution is negative.

Step 2. Factor the fraction.

$$\frac{2*2*5}{5*2*2*3}$$

Step 3. Combine and reduce like terms.

$$\left(\frac{2}{2}\right)\left(\frac{2}{2}\right)\left(\frac{5}{5}\right)\left(\frac{1}{3}\right) =$$

$$\mathbf{1} * \mathbf{1} * \mathbf{1} * \left(\frac{1}{3}\right)$$

Step 4. Multiply.

$$\mathbf{1} * \mathbf{1} * \mathbf{1} * \left(\frac{1}{3}\right) = \frac{1}{3}$$

$-\frac{1}{3}$ **This is the solution.**

(Recall that step 1 determined that the solution would be negative.)

Example 3. Simplify $\frac{15axy + 15ay + 30xy + 30y}{21x + 21}$ **.**

Step 1. Determine the sign of the solution.

The solution will be positive because the numerator and denominator are both positive.

Step 2. Factor the fraction. (This example uses factoring by grouping.)

$$\frac{15axy + 15ay + 30xy + 30y}{21x + 21}$$

$$= \frac{(15axy + 15ay) + (30xy + 30y)}{21(x+1)}$$

$$= \frac{15ay(x+1) + 30y(x+1)}{3 * 7 * (x+1)}$$

$$= \frac{(x+1)(15ay + 30y)}{3 * 7 * (x+1)}$$

$$= \frac{(x+1)(15y)(a+2)}{3 * 7 * (x+1)}$$

$$= \frac{3 * 5 * y * (x+1)(a+2)}{3 * 7 * (x+1)}$$

Step 3. Combine like terms and reduce.

$$\left(\frac{3}{3}\right) \left(\frac{5}{7}\right) \left(\frac{x+1}{x+1}\right) \left(\frac{y}{1}\right) \left(\frac{a+2}{1}\right)$$

$$\mathbf{1} * \left(\frac{5}{7}\right) * \mathbf{1} * \left(\frac{y}{1}\right) * \left(\frac{a+2}{1}\right)$$

Step 4. Multiply.

$$\mathbf{1} * \left(\frac{5}{7}\right) * \mathbf{1} * \left(\frac{y}{1}\right) * \left(\frac{a+2}{1}\right)$$

$$= \frac{5y(a+2)}{7}$$ **This is the final solution.**

When dividing rational expressions the divisor is first inverted, then the procedure used for multiplying is followed. The definition for dividing rational expressions is :

$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} * \frac{d}{c}$ **, where a, b, c and d are integers *<u>and</u>* b, c, d $\neq$ 0.**

Note: $\frac{c}{d}$ **and** $\frac{d}{c}$ **are known as *reciprocals or multiplicative inverses*.**

Example 4. **Perform the following operation and reduce the solution.**

$$\frac{3}{5} \div \frac{3}{10}$$

Step 1. **Determine the sign of the solution.**

Since both terms are positive, the solution will be positive.

Step 2. **Invert the divisor.**

$$\frac{3}{5} \div \frac{3}{10} = \left(\frac{3}{5}\right)\left(\frac{10}{3}\right)$$

Step 3. **Factor.**

$$\left(\frac{3}{5}\right)\left(\frac{10}{3}\right) = \frac{3*2*5}{3*5}$$

Step 4. **Combine like factors and reduce.**

$$\frac{3*2*5}{3*5} = \left(\frac{3}{3}\right)\left(\frac{5}{5}\right)\left(\frac{2}{1}\right)$$

$$= 1 * 1 * 2$$

Step 5. **Multiply.**

$1 * 1 * 2 = 2$ **This is the solution of the problem.**

Example 5. Divide and simplify : $\frac{a^2+7a+12}{b^2-b-2} \div \frac{a^2+2a-8}{b^2+b-6}$

Step 1. Determine the sign of the solution.

Since both terms are positive, the solution is positive.

Step 2. Invert the divisor.

$$\frac{a^2+7a+12}{b^2-b-2} \div \frac{a^2+2a-8}{b^2+b-6} = \left(\frac{a^2+7a+12}{b^2-b-2}\right)\left(\frac{b^2+b-6}{a^2+2a-8}\right)$$

Step 3. Factor.

$$\left(\frac{a^2+7a+12}{b^2-b-2}\right)\left(\frac{b^2+b-6}{a^2+2a-8}\right) = \frac{(a+3)(a+4)(b-2)(b+3)}{(b+1)(b-2)(a+4)(a-2)}$$

Step 4. Group like factors and reduce.

$$\frac{(a+3)(a+4)(b-2)(b+3)}{(b+1)(b-2)(a+4)(a-2)} = \left(\frac{a+3}{a-2}\right)\left(\frac{a+4}{a+4}\right)\left(\frac{b-2}{b-2}\right)\left(\frac{b+3}{b+1}\right)$$

$$= \left(\frac{a+3}{a-2}\right)(1)\,(1)\left(\frac{b+3}{b+1}\right)$$

Step 5. Multiply.

$$\left(\frac{a+3}{a-2}\right)(1)\,(1)\left(\frac{b+3}{b+1}\right) = \frac{(a+3)(b+3)}{(a-2)(b+1)}$$ **This is the solution.**