RATIONAL EQUATION: APPLICATIONS & PROBLEM SOLVING

When finding the LCD of a problem involving the addition or subtraction of fractions, it may be necessary to factor some denominators to discover some restricted values, that is values that if used make the denominator of one or more of the fractional terms zero.

Example 1. **Solve** $\frac{a+12}{a^2-16}-\frac{3}{a-4}=\frac{1}{a+4}$

Step 1. Factor the denominators and find the LCD.

$$\frac{a+12}{(a+4)(a-4)}-\frac{3}{a-4}=\frac{1}{a+4}$$ **LCD = (a + 4) (a – 4)**

a $\neq \pm 4$

Step 2. Multiply all terms by the LCD.

$$(a+4)(a-4)\left(\frac{a+12}{(a+4)(a-4)}-\frac{3}{a-4}\right)=\left(\frac{1}{a+4}\right)(a+4)(a-4)$$

$$(a+4)(a-4)\left(\frac{a+12}{(a+4)(a-4)}\right)-\left[\left(\frac{3}{a-4}\right)(a+4)(a-4)\right]=\left(\frac{1}{a+4}\right)(a+4)(a-4)$$

Step 3. Simplify.

$$(a+4)(a-4)\left(\frac{a+12}{(a+4)(a-4)}\right)-\left[\left(\frac{3}{a-4}\right)(a+4)(a-4)\right]=\left(\frac{1}{a+4}\right)(a+4)(a-4)$$

$$(a+12)-[3(a+4)]=1(a-4)$$

$$a+12-[3a+12]=a-4$$

$$a + 12 - 3a - 12 = a - 4$$

$$-2a = a - 4$$

Step 4. Solve for a.

-2a = a – 4

-2a + 2a = a – 4 + 2a

0 = 3a – 4

0 + 4 = 3a – 4 + 4

4 = 3a

$$\frac{4}{3} = \frac{3a}{3}$$

$$\frac{4}{3} = a$$

Example 2. Solve $\frac{2}{p} - \frac{3}{p-2} = -\frac{3}{p^2 - 2p}$ **for p.**

Step 1. Find the LCD and restricted values.

LCD = (p) (p – 2)

$p \neq 0$, 2

Step 2. Multiply all terms by the LCD.

$$(p)(p-2)\left(\frac{2}{p} - \frac{3}{p-2}\right) = \left(\frac{-3}{p^2 - 2p}\right)(p)(p-2)$$

$$(p)(p-2)\left(\frac{2}{p}\right) - \left[\left(\frac{3}{p-2}\right)(p)(p-2)\right] = \left(\frac{-3}{p^2 - 2p}\right)(p)(p-2)$$

Step 3. Simplify.

(2) (p – 2) - [(3) (p)] = - 3

2p – 4 – 3p = - 3

- p – 4 = - 3

Step 4. Solve for p.

- p – 4 + p = - 3 + p

- 4 = - 3 + p

- 4 + 3 = - 3 + p + 3

- 1 = p

This is the solution since this value was not restricted and the check works.

Using the principles covered, it is now possible to solve for a single unknown in a formula.

Example 3. **Solve** $I = \dfrac{nE}{R + nr}$ **for n.**

Step 1. **Alter the formula so that there are no fractions. To do this multiply both sides by the denominator.**

$$I(R + nr) = \frac{nE}{R + nr}(R + nr)$$

$$RI + nrI = nE$$

Step 2. **Group together all terms that contain the variable to be solved.**

$$RI + nrI = nE$$

$$RI + nrI - nrI = nE - nrI$$

$$RI = nE - nrI$$

Step 3. **Factor out the variable to be solved.**

$$RI = nE - nrI$$

$$RI = n (E - rI)$$

Step 4. **Isolate n.**

$$RI = n (E - rI)$$

$$\frac{RI}{E - rI} = \frac{n(E - rI)}{(E - rI)}$$

$$\frac{RI}{E - rI} = n$$

This is the solution because there is only one of the variables, n , that was to be solved for.

Problems involving uniform motion and time may also be solved using these techniques.

Example 4. **Frank can clean a certain vacant lot in 7 hours while Sue can do it in 5 hours. How long will it take them both to clean the lot?**

Step 1. Organize the data and create a formula.

a.) Let x represent the number of hours it takes both to do the task.

b.) Frank completes $\frac{1}{7}$ of the job an hour.

c.) Sue completes $\frac{1}{5}$ of the job an hour.

d.) Together they complete $\frac{1}{x}$ of the job an hour.

Therefore: The part done by Frank + The part done by Sue = The whole job is

$$\frac{1}{7} \quad + \quad \frac{1}{5} \quad = \quad \frac{1}{x}$$

Step 2. Find the LCD and restrictions.

LCD = 7 * 5 * x = 35x
$x \neq 0$

Step 3. Multiply all terms by the LCD and simplify.

$$35x\left(\frac{1}{7}+\frac{1}{5}\right)=35x\left(\frac{1}{x}\right)$$

$$35x\left(\frac{1}{7}\right)+35x\left(\frac{1}{5}\right)=35x\left(\frac{1}{x}\right)$$

$5x + 7x = 35$

$12x = 35$

Step 4. Solve for x and convert.

$12x = 35$

$$\frac{12x}{12}=\frac{35}{12}$$

$x = \frac{35}{12}$ hours. $\frac{35}{12}$ hours = $2\ \frac{11}{12}$ hours.

Since $\frac{1}{12}$ hour is 5 minutes the total converted time would be 2 hours and 55 minutes.

Example 5. Gerald drives 300 miles east to Houston. He averages 70 miles per hour on the interstate, but while going through Speed Trap County he reduces his speed to 55 miles per hour. If the trip takes 5 hours how many miles of the road were in Speed Trap County?

Step 1. Organize the data.

a.) Let x represent the number of miles in Speed Trap County (at reduced speed).

b.) The balance of the miles driven at 70 mph would be 300 – x .

Step 2. Determine the formula to be used to solve the problem.

The formula D = RT, where D= distance, R = rate and T = time, will be used. Since the time is what is unknown, the equation is altered to solve for T.

D = RT

$$\frac{D}{R} = \frac{RT}{R}$$

$$\frac{D}{R} = T$$

Step 3. Substitute the values found for the formula variables.

Location	D	R	T
Outside of Speed Trap County	300 – x	70	$\frac{300-x}{70}$
Inside of Speed Trap County	x	55	$\frac{x}{55}$

Step 4. Create the formula to solve the problem.

Since the Time at 70 mph + Time at 60 mph = 5 hours (total travel time)

The formula is $\frac{300-x}{70} + \frac{x}{55}$ = 5 hours

Step 5 Find the LCD and multiply all terms by it.

LCD = 2 * 5 * 7 * 11 = 770

$$770\left(\frac{300-x}{70}+\frac{x}{55}\right) = 770\,(\,5\,)$$

$$770\left(\frac{300-x}{70}\right) + 770\left(\frac{x}{55}\right) = 770\,(\,5\,)$$

Step 6. Simplify.

11 (300 – x) + 14 (x) = 3850

3300 – 11x + 14x = 3850

3300 + 3x = 3850

3300 + 3x – 3300 = 3850 – 3300

3x = 550

Step 7. Solve for x.

3x = 550

$$\frac{3x}{3}=\frac{550}{3}$$

x = 183 $\frac{1}{3}$ miles.