Review Exercise Set 14

Exercise 1: Bill drives 270 miles in the same amount of time it takes Susan to drive 250 miles. If Bill averages 4 miles per hour faster than Susan, find their rates.

Exercise 2: One number is three times as large as another. The sum of their reciprocals is four. Determine the numbers.

Exercise 3: It takes one train 2 hours longer to travel 350 miles than a second train. The rate of the second train is 20 miles per hour faster than the rate of the first train. Find the rates of both trains.

Exercise 4: The sum of two numbers is eighty. If the larger number is divided by the smaller number, the quotient is seven with a remainder of eight. Find the numbers.

Exercise 5: Pete can do a certain task in 3 hours. Jon can perform the same task but in 5 hours. If Pete helps Jon perform this task, how long will it take them to do it working together?

Review Exercise Set 14 Answer Key

Exercise 1: Bill drives 270 miles in the same amount of time it takes Susan to drive 250 miles. If Bill averages 4 miles per hour faster than Susan, find their rates.

Setup a table with the given information

	Distance	Rate	Time
Bill	270 miles	r + 4	$\frac{270}{r+4}$
Susan	250 miles	r	$\frac{250}{r}$

Setup the equation

Since we are told that Bill and Susan drive for the same amount of time, we can set their times equation to each other and then solve for r.

$$\frac{270}{r+4} = \frac{250}{r}$$

Use the cross multiplication property and solve for r

270 * r = 250 * (r + 4)
270r = 250r + 1000
270r - 250r = 1000
20r = 1000
r = 50

Now, substitute r into the expression representing Bill's rate

r + 4 = 50 + 4 = 54

Bill's rate was 54 miles per hour and Susan's was 50 miles per hour.

Exercise 2: One number is three times as large as another. The sum of their reciprocals is four. Determine the numbers.

Assign variable expressions to the two numbers

x = smaller number
3x = larger number

$\frac{1}{x}$ = reciprocal of smaller number

$\frac{1}{3x}$ = reciprocal of larger number

Setup the equation and solve for x

$$\frac{1}{x}+\frac{1}{3x}=4$$

$$\left[\frac{1}{x}+\frac{1}{3x}\right]\times 3x=4\times 3x$$

$$\frac{1}{x}\times 3x+\frac{1}{3x}\times 3x=4\times 3x$$

$$3+1=12x$$

$$4=12x$$

$$\frac{4}{12}=x$$

$$\frac{1}{3}=x$$

Substitute x into the expression for the larger number

$$3x=3\times\frac{1}{3}=1$$

The two numbers are $\frac{1}{3}$ and 1.

Exercise 3: It takes one train 2 hours longer to travel 350 miles than a second train. The rate of the second train is 20 miles per hour faster than the rate of the first train. Find the rates of both trains.

Setup a table with the given information

	Distance	Time	Rate
1st train	350 miles	t + 2	$\frac{350}{t+2}$
2nd train	350 miles	t	$\frac{350}{t}$

Setup the equation

Since we are told that the 2nd train was traveling 20 miles per hour faster than the 1st train, we can set up the equation by adding 20 to the rate of the 1st train and then setting it equal to the 2nd train.

$$\frac{350}{t+2}+20=\frac{350}{t}$$

Multiply the equation by the LCD and solve for t

$$\left[\frac{350}{t+2}+20\right]\times t(t+2)=\left[\frac{350}{t}\right]\times t(t+2)$$

$$\left[\frac{350}{t+2}\right]\times t(t+2)+[20]\times t(t+2)=\left[\frac{350}{t}\right]\times t(t+2)$$

$$350\times t+20\times\left(t^2+2t\right)=350\times(t+2)$$

$$350t+20t^2+40t=350t+700$$

$$20t^2+390t=350t+700$$

$$20t^2+390t-350t-700=0$$

$$20t^2+40t-700=0$$

$$20\left(t^2+2t-35\right)=0$$

$$20(t-5)(t+7)=0$$

$$t-5=0 \quad or \quad t+7=0$$

$$t=5 \qquad\qquad t=-7$$

Time cannot be negative, so t must equal 5.

Exercise 3 (Continued):

Use the value of t to solve for the rates of the two trains

$$\frac{350}{t+2}=\frac{350}{5+2}=\frac{350}{7}=50$$
$$\frac{350}{t}=\frac{350}{5}=70$$

The 1st train's rate was 50 miles per hour and the 2nd train's rate was 70 miles per hour.

Exercise 4: The sum of two numbers is eighty. If the larger number is divided by the smaller number, the quotient is seven with a remainder of eight. Find the numbers.

Assign variable expressions to the two numbers

x = smaller number
80 - x = larger number

(Since the two numbers must add up to 80 the larger number must be equal to 80 minus the smaller number which is x)

$\frac{80-x}{x}$ = larger number divided by smaller number

Setup the equation

We are told that the remainder is 8, so when we write the equation we must place the remainder over the divisor of x and then add it to the quotient of 7.

$$\frac{80-x}{x}=7+\frac{8}{x}$$

Multiply the equation by the LCD and solve for x

$$\frac{80-x}{x}\times x=\left[7+\frac{8}{x}\right]\times x$$
$$\frac{80-x}{x}\times x=[7]\times x+\left[\frac{8}{x}\right]\times x$$
$$80-x=7x+8$$
$$80-8=7x+x$$
$$72=8x$$
$$9=x$$

Exercise 4 (Continued):

Use the value of x to find the larger number

80 - x = 80 - 9 = 71

The smaller number is 9 and the larger number is 71.

Exercise 5: Pete can do a certain task in 3 hours. Jon can perform the same task but in 5 hours. If Pete helps Jon perform this task, how long will it take them to do it working together?

Assign variable expressions to the two numbers

$\frac{1}{3}$ = Pete's rate

$\frac{1}{5}$ = Jon's rate

$\frac{1}{x}$ = Combined rate

Setup the equation

Pete's rate + Jon's rate = Combined rate

$$\frac{1}{3}+\frac{1}{5}=\frac{1}{x}$$

Multiply the equation by the LCD and solve for x

$$\left[\frac{1}{3}+\frac{1}{5}\right]\times 15x=\left[\frac{1}{x}\right]\times 15x$$

$$\left[\frac{1}{3}\right]\times 15x+\left[\frac{1}{5}\right]\times 15x=\left[\frac{1}{x}\right]\times 15x$$

$$5x+3x=15$$

$$8x=15$$

$$x=\frac{15}{8}$$

$$x=1\frac{7}{8}$$

Together it would take them $1\frac{7}{8}$ hours (or approximately 1 hour 53 minutes) to complete the task.