SIMPLIFYING RATIONAL EXPRESSIONS

Recall that any number that can be written in the form $\frac{A}{B}$, where A and B are integers and B $\neq$ 0 is a rational number. Some examples of rational numbers are:

$\frac{1}{2}$, $\frac{-2}{3}$, $\frac{3}{-4}$, $-\frac{1}{5}$, **0.3 =** $\frac{3}{10}$, **0 =** $\frac{0}{1}$ **or** $\frac{0}{2}$ **etc. , 5 =** $\frac{5}{1}$ **or** $\frac{-5}{-1}$,

- 6 = $\frac{-6}{1}$ **or** $\frac{6}{-1}$ **, 3** $\frac{1}{3}$ **=** $\frac{10}{3}$ **.**

The fundamental principle of fractions allows one to reduce a fraction by removing nonzero factors that are present in *both* the numerator and denominator because they are really a fractional form of 1.

$$\frac{a*k}{b*k} = \frac{a}{b}*\frac{k}{k} = \frac{a}{b}*1 = \frac{a}{b}$$

Note: Though a commonly used term, *cancel*, is not appropriate because, as shown in the principle, nothing "disappears" but rather is recognized to really be 1. Recall that a value multiplied or divided by 1 is still that value.

Example 1. Reduce $\frac{6}{10}$ to lowest terms.

Step 1. Prime factor the fraction.

$$\frac{6}{10} = \frac{2*3}{2*5}$$

Step 2. Apply the principle and group terms.

$$\frac{2*3}{2*5} = \left(\frac{2}{2}\right)\left(\frac{3}{5}\right)$$

Step 3. Simplify.

$$\left(\frac{2}{2}\right)\left(\frac{3}{5}\right) =$$

$$\mathbf{1} * \frac{3}{5} =$$

$\frac{3}{5}$ **This is the solution to the problem.**

The quotient of two polynomials is known as a rational expression.

Example 2. **Reduce** $\frac{30xy^2z}{42zxy}$ **to lowest terms.**

Step 1. Prime factor the fraction.

$$\frac{2*3*5*x*y*y*z}{2*3*7*x*y*a*1}$$

(Note that a 1 was included in the factors of the denominator. This was done to make the number of factors in the numerator and the denominator the same.)

Step 2. Group the factors of the numerator and the denominator.

$$\left(\frac{2}{2}\right)\left(\frac{3}{3}\right)\left(\frac{5}{7}\right)\left(\frac{x}{x}\right)\left(\frac{y}{y}\right)\left(\frac{y}{a}\right)\left(\frac{z}{1}\right)$$

Step 3. Simplify.

$$\mathbf{(1)}\ \mathbf{(1)}\left(\frac{5}{7}\right)\mathbf{(1)}\,\mathbf{(1)}\left(\frac{y}{a}\right)\left(\frac{z}{1}\right)$$

Step 4. Continue.

$\frac{5yz}{7a}$ **This is solution to the problem.**

Example 3. **Reduce** $\frac{x^2+x-2}{x^2-4x+3}$ **to lowest terms.**

Step 1. Prime factor the fraction.

$$\frac{x^2+x-2}{x^2-4x+3} = \frac{(x+2)(x-1)}{(x-1)(x-3)}$$

Step 2. Group like terms.

$$\frac{(x+2)(x-1)}{(x-1)(x-3)} = \frac{(x-1)}{(x-1)} * \frac{(x+2)}{(x-3)}$$

Step 3. Simplify.

$$\frac{(x-1)}{(x-1)} * \frac{(x+2)}{(x-3)} =$$

$$\mathbf{1} * \frac{x+2}{x-3} =$$

$\frac{x+2}{x-3}$ **This is the solution to the problem.**

Example 4. **Reduce** $\frac{x^2+x-2}{x-1}$ **to lowest terms.**

Step 1. Prime factor the fraction.

$$\frac{x^2+x-2}{x-1} = \frac{(x+2)(x-1)}{(x-1)}$$

Step 2. Group the common factors.

$$\frac{(x+2)(x-1)}{(x-1)} = \frac{(x-1)}{(x-1)} * \frac{(x+2)}{1}$$

Step 3. Simplify.

$$\frac{(x-1)}{(x-1)} * \frac{(x+2)}{1} =$$

1 * (x + 2) =

(x + 2)

This is the solution to the problem. Note that a factor of one was included in the denominator to make the number of factors in the numerator and denominator the same.

Example 5. Reduce $\frac{x-1}{x^2+x-2}$ **to lowest terms.**

Step 1. Prime factor the fraction.

$$\frac{x-1}{x^2+x-2} = \frac{x-1}{(x-2)(x-1)}$$

Step 2. Group the like terms.

$$\frac{x-1}{(x-2)(x-1)} = \frac{(x-1)}{(x-1)} * \frac{1}{(x-2)}$$

Step 3. Simplify.

$$\frac{(x-1)}{(x-1)} * \frac{1}{(x-2)} =$$

$$\mathbf{1} * \frac{1}{(x-2)} =$$

$\frac{1}{(x-2)}$ **This is the solution to the problem.**

Note if the factors being divided are opposites of each other the solution will be – 1 .

Example 6. $\frac{a}{-a}$ **or** $\frac{-a}{a}$ **= -1 ,** $\frac{a-b}{b-a}$ **or** $\frac{b-a}{a-b}$ **= -1**