

## SOLVING RATIONAL EQUATIONS

Recall that when solving for fractions that are added or subtracted from each other that a common denominator must be found. This is true even if the denominator contains a variable. An extra condition that must always be considered is that any value of the variable that makes the denominator zero must be excluded.

**Example 1.** Solve  $\frac{2}{a} + \frac{3}{8} = \frac{14}{a}$

**Step 1. Find the LCD.**

The LCD for this problem is  $8a$ . Also  $a \neq 0$ .

**Step 2. Multiply all terms by the LCD.**

$$(8a) \left( \frac{2}{a} + \frac{3}{8} \right) = \left( \frac{14}{a} \right) (8a)$$

$$(8a) \left( \frac{2}{a} \right) + (8a) \left( \frac{3}{8} \right) = \left( \frac{14}{a} \right) (8a)$$

**Step 3. Simplify.**

$$(8a) \left( \frac{2}{a} \right) + (8a) \left( \frac{3}{8} \right) = \left( \frac{14}{a} \right) (8a)$$

$$8 * 2 + a * 3 = 14 * 8$$

$$16 + 3a = 112$$

**Step 4. Solve for a.**

$$16 + 3a = 112$$

$$16 + 3a - 16 = 112 - 16$$

$$3a = 96$$

$$\frac{3a}{3} = \frac{96}{3}$$

$$a = 32$$

**This is the solution since  $a \neq 0$  and 32 yields a valid check.**

**Example 2.** Solve  $\frac{45-x}{x} = 9 - \frac{5}{x}$  ;  $x \neq 0$  .

**Step 1. Find the LCD.**

**The LCD for this problem is  $x$  .**

**Step 2. Multiply all terms by the LCD of  $x$ .**

$$x \left( \frac{45-x}{x} \right) = \left( 9 - \frac{5}{x} \right) x$$

$$(x) \left( \frac{45-x}{x} \right) = (9)(x) - \left( \frac{5}{x} \right) (x)$$

**Step 3. Simplify.**

$$(x) \left( \frac{45-x}{x} \right) = (9)(x) - \left( \frac{5}{x} \right) (x)$$

$$45 - x = 9x - 5$$

**Step 4. Solve for  $x$ .**

$$45 - x = 9x - 5$$

$$45 - x + x = 9x - 5 + x$$

$$45 = 10x - 5$$

$$45 + 5 = 10x - 5 + 5$$

$$50 = 10x$$

$$\frac{50}{10} = \frac{10x}{10}$$

$$5 = x$$

**This is the solution since  $x \neq 0$  and 5 yields a valid check.**

**Example 3.** Solve  $\frac{3}{b-4} = \frac{4}{b+3}$ ;  $b \neq -3$  or  $4$

**Step 1. Find the LCD.**

The LCD for this problem is  $(b - 4)(b + 3)$ .

**Step 2. Multiply all terms by the LCD.**

$$(b - 4)(b + 3) \left( \frac{3}{b-4} \right) = \left( \frac{4}{b+3} \right) (b - 4)(b + 3)$$

**Step 3. Simplify.**

$$(b - 4)(b + 3) \left( \frac{3}{b-4} \right) = \left( \frac{4}{b+3} \right) (b - 4)(b + 3)$$

$$(b + 3)(3) = (4)(b - 4)$$

$$3b + 9 = 4b - 16$$

**Step 4. Solve for b.**

$$3b + 9 = 4b - 16$$

$$3b + 9 - 3b = 4b - 16 - 3b$$

$$9 = b - 16$$

$$9 + 16 = b - 16 + 16$$

$$25 = b$$

**This is the solution since  $b \neq -3$  or  $4$  and  $25$  yields a valid check.**

**Example 4.** Solve  $\frac{x}{x-2} + \frac{2}{5} = \frac{2}{x-2}$  ;  $x \neq 2$

**Step 1. Find the LCD.**

The LCD for this problem is  $5(x - 2)$ .

**Step 2. Multiply all terms by the LCD.**

$$(5)(x-2) \left( \frac{x}{x-2} + \frac{2}{5} \right) = \left( \frac{2}{x-2} \right) (5)(x-2)$$

$$(5)(x-2) \left( \frac{x}{x-2} \right) + (5)(x-2) \left( \frac{2}{5} \right) = \left( \frac{2}{x-2} \right) (5)(x-2)$$

**Step 3. Simplify**

$$(5)(x-2) \left( \frac{x}{x-2} \right) + (5)(x-2) \left( \frac{2}{5} \right) = \left( \frac{2}{x-2} \right) (5)(x-2)$$

$$(5)(x) + (x-2)(2) = (2)(5)$$

$$5x + 2x - 4 = 10$$

$$7x - 4 = 10$$

**Step 4. Solve for x.**

$$7x - 4 = 10$$

$$7x - 4 + 4 = 10 + 4$$

$$7x = 14$$

$$\frac{7x}{7} = \frac{14}{7}$$

$$x = 2$$

This is not the solution since 2 was excluded at the beginning of the problem. This problem then has no real solutions.

Where two numbers are compared using division, a ratio is said to exist. When two ratios are made equal to each other a proportion exists. These ideas lead to the cross-multiplication property of proportions, which states:

$$\text{If } \frac{a}{b} = \frac{c}{d} \text{ ( } b \neq 0 \text{ and } d \neq 0 \text{ ) then } ad = bc$$

**Example 5.** Using cross-multiplication, solve  $\frac{3}{x-2} = \frac{5}{x+4}$  ;  $x \neq 2, -4$ .

**Step 1.** Apply the property to the problem.

$$\frac{3}{x-2} = \frac{5}{x+4}$$

$$3(x+4) = 5(x-2)$$

**Step 2.** Simplify.

$$3(x+4) = 5(x-2)$$

$$3x + 12 = 5x - 10$$

**Step 3.** Solve for x.

$$3x + 12 = 5x - 10$$

$$3x + 12 - 3x = 5x - 10 - 3x$$

$$12 = 2x - 10$$

$$12 + 10 = 2x - 10 + 10$$

$$22 = 2x$$

$$\frac{22}{2} = \frac{2x}{2}$$

$$11 = x$$

**This is the solution since  $x \neq 2$  or  $-4$  and 11 yields a valid check.**