ADDING, SUBTRACTING, AND MULTIPLYING RADICALS

Use of the distributive property demonstrates that when expressions that contain radicals are added or subtracted from one another that they must have the same radicand and index number.

Example 1. **5x + 4x = (5 + 4) (x) = 9x**

$$5\sqrt[3]{9}+4\sqrt[3]{9}=(5+4)\left(\sqrt[3]{9}\right)=9\sqrt[3]{9}$$

Where the radicands are not already the same, factoring may be required before the distributive property is applied.

Example 2. **Simplify** $9\sqrt{3r}-2\sqrt{12r}+5\sqrt{27r}$ **.**

Step 1. Factor the radicands.

$$9\sqrt{3r}-2\sqrt{12r}+5\sqrt{27r}$$

$$9\sqrt{3r}-2\sqrt{4*3r}+5\sqrt{9*3r}$$

Step 2. Simplify.

$$9\sqrt{3r}-2\sqrt{4*3r}+5\sqrt{9*3r}$$

$$9\sqrt{3r}-2\sqrt{4}*\sqrt{3r}+5\sqrt{9}*\sqrt{3r}$$

$$9\sqrt{3r}-2*2*\sqrt{3r}+5*3*\sqrt{3r}$$

$$9\sqrt{3r}-4\sqrt{3r}+15\sqrt{3r}$$

Step 3. Distribute and solve.

$$9\sqrt{3r}-4\sqrt{3r}+15\sqrt{3r}$$

$$(9-4+15)\sqrt{3r}$$

$$20\sqrt{3r}$$

This is the solution.

Recall the property that allows the multiplication of radicals with similar index numbers:

$$\sqrt[n]{bc} = \sqrt[n]{b} * \sqrt[n]{c} \quad \textbf{or} \quad \sqrt[n]{b} * \sqrt[n]{c} = \sqrt[n]{bc}$$

Example 3. Multiply and simplify.

a.) $\mathbf{3}\ \sqrt{5}\ *\ \mathbf{2}\ \sqrt{2}$

$= (3*2)\ \left(\sqrt{5}*\sqrt{2}\right)$

$= \mathbf{6}\ \sqrt{10}$

b.) $\mathbf{4}\ \sqrt{6}\ *\ \mathbf{2}\ \sqrt{2}$

$= (4*2)\ \left(\sqrt{6}*\sqrt{2}\right)$

$= \mathbf{8}\ \sqrt{12}\ = \mathbf{8}\ \sqrt{4*3}\ = \mathbf{8}\ \sqrt{4}*\sqrt{3}$

$= \mathbf{8} * \mathbf{2} * \sqrt{3}\ = \mathbf{16}\ \sqrt{3}$

The distributive property applies when multiplying equations with radicals in polynomials.

$a(b+c) = ab + ac$, $a(b+c+d) = ab + ac + ad$, $(a+b)(c-d) = ac - ad + bc - bd$

Example 4. Multiply and simplify.

a.) $\sqrt{k}\ \left(\sqrt{a} + \sqrt{b}\right) = \sqrt{ak} + \sqrt{bk}$

b.) $\left(\sqrt{a} + \sqrt{b}\right)\left(\sqrt{c} - \sqrt{d}\right) = \sqrt{ac} - \sqrt{ad} + \sqrt{bc} - \sqrt{bd}$

c.) $\left(\sqrt{10} + 3\right)\left(\sqrt{10} - 3\right) = \sqrt{100} - 3\sqrt{10} + 3\sqrt{10} - 9$

$= \mathbf{10 - 9}$

$= \mathbf{1}$

d.) $\left(\sqrt{7} + 3\right)\left(\sqrt{7} + 3\right) = \sqrt{49} + 3\sqrt{7} + 3\sqrt{7} + 9$

$= \mathbf{7 + 6}\ \sqrt{7}\ \mathbf{+ 9}$

$= \mathbf{16 + 6}\sqrt{7}$