COMPLEX NUMBERS

By definition a complex number is one which can be expressed in the form of $a + bi$. In the relationship of numbers, all real numbers are a subset of the set of complex numbers. The number i is used to solve certain equations that cannot be solved using the set of real numbers. In using i it is understood that $i^2 = -1$, that is, i is a number whose square is –1 . Keeping in mind that solutions involving complex numbers must be in the form of $a + bi$ the following examples will demonstrate how to perform some basic operations involving complex numbers.

Example 1. Add.

a.) $(2 + 3i) + (6 + 4i)$

Step 1. Group like terms.

$(2 + 3i) + (6 + 4i) = 2 + 3i + 6 + 4i$

$= (2 + 6) + (3i + 4i)$

Step 2. Factor the *i* from its group.

$(2 + 6) + (3i + 4i) = 8 + (3 + 4)i$

$= 8 + 7i$

b.) $5 + (9 - 3i)$

Step 1. Group like terms. (Note 5 may be expressed as $5 + 0i$)

$5 + (9 - 3i) = 5 + 0i + 9 - 3i$

$= (5 + 9) + (0i - 3i)$

Step 2. Factor the *i* from its group.

$(5 + 9) + (0i - 3i) = 14 + (0 - 3)i$

$= 14 + (-3)i$ or $14 - 3i$

Example 2. Subtract.

a.) **(6 + 5*i*) – (3 + 2*i*)**

Step 1. Group like terms.

(6 + 5*i*) – (3 + 2*i*)	**= 6 + 5*i* - 3 – 2*i***
	= 6 + 5*i* + (-3) + (-2*i*)
	= (6 + (-3)) + (5*i* + (-2*i*))
	= (6 – 3) + (5*i* – 2*i*)

Step 2. Factor the *i* from its group.

(6 – 3) + (5*i* – 2*i*)	**= 3 + (5 – 2)*i***
	= 3 + 3*i*

b.) **(7 – 3*i*) – (8 – 6*i*)**

Step 1. Group like terms.

(7 – 3*i*) – (8 – 6*i*)	**= 7 – 3*i* – 8 + 6*i***
	= 7 + (-3*i*) + (-8) + 6*i*
	= (7 + (-8)) + ((-3*i*) + 6*i*)

Step 2. Factor the *i* from its group.

(7 + (-8)) + ((-3*i*) + 6*i*)	**= -1 + (-3 + 6)*i***
	= -1 + 3*i*

Dealing with complex numbers that one tries to find a product or quotient of, demands that one understand that since $i^2 = -1$, $i = \sqrt{-1}$. The definition $\sqrt{-b} = i\sqrt{b}$ is used to resolve the square root of a negative radicand. Note that the resolved term is $i\sqrt{b}$ instead of $\sqrt{b}\ i$ so that it will not be mistakenly written as $\sqrt{bi}$.

Example 3. Simplify.

a.) $\sqrt{-100} = \sqrt{100}\ \sqrt{-1} = \sqrt{100}\ i = 10i$

b.) $\sqrt{-2} = \sqrt{2}\ \sqrt{-1} = \sqrt{2}\ i = i\sqrt{2}$

Example 4. **Multiply.**

a.) $\sqrt{-3}\sqrt{-7} = i\sqrt{3}\ i\sqrt{7} = i^2\sqrt{21} = (-1)\sqrt{21} = -\sqrt{21}$

b.) $\sqrt{-2}\sqrt{-8} = i\sqrt{2}\ i\sqrt{8} = i^2\sqrt{16} = (-1)(4) = -4$

Example 5. **Simplify.**

a.) $(3 + 5i)(4 - 2i)$
$= 12 - 6i + 20i - 10i^2$
$= 12 + 14i - [10(-1)]$
$= 12 + 14i - [-10]$
$= 12 + 14i + 10$
$= 22 + 14i$

b.) $(2 + 3i)(1 - 5i)$
$= 2 - 10i + 3i - 15i^2$
$= 2 - 7i - [15(-1)]$
$= 2 - 7i - [-15]$
$= 2 - 7i + 15$
$= 17 - 7i$ **or** $17 + (-7i)$

Note that when $a + bi$ **is multiplied by its conjugate,** $a - bi$**, the product is** $a^2 + b^2$ **.**

Example 6. **Find the quotient.**

a.) $\dfrac{4 - 3i}{5 + 2i}$

Step 1. Multiply the problem by its conjugate. (This is done to resolve the *i* values in the denominator.)

$$\frac{4 - 3i}{5 + 2i}\left(\frac{5 - 2i}{5 - 2i}\right) = \frac{20 - 8i - 15i + 6i^2}{5^2 + 2^2}$$

$$= \frac{20 - 23i + 6i^2}{25 + 4}$$

Step 2. Simplify.

$$\frac{20 - 23i + 6i^2}{25 + 4} = \frac{20 - 23i + [(6)(-1)]}{29}$$

$$= \frac{20 - 23i + (-6)}{29}$$

$$= \frac{14 - 23i}{29}$$

$$= \frac{14}{29} - \frac{23i}{29} \textbf{ or } \frac{14}{29} + \left(-\frac{23i}{29}\right)$$

b.) $\frac{1 + i}{i}$

Step 1. Multiply the problem by its conjugate.

$$\frac{1 + i}{i}\left(\frac{-i}{-i}\right) = \frac{-i - i^2}{-i^2}$$

Step 2. Simplify.

$$\frac{-i - i^2}{-i^2} = \frac{-i - (-1)}{-(-1)}$$

$$= \frac{-i + 1}{1}$$

$$= (-i) + 1$$

$$= 1 + (-i)$$