Dividing Radicals

In previous sections we learned about rational expressions. In some instances there will be a radical in the denominator of a rational expression. When this situation occurs it is necessary to perform an operation called "rationalizing the denominator." Rationalizing the denominator will remove the radical from the denominator by multiplying the numerator and denominator of the rational expression by a radical that will make the radicand in the denominator a perfect power.

Example 1: Simplify $\frac{3}{x\sqrt{2}}$

Solution:

Step 1: Multiply the numerator and denominator by a radical that makes the radicand in the denominator a perfect power.

The radicand in the denominator is 2 and the index for the radical is 2
So the lowest perfect power would be 2^2

To obtain 2^2 in the radicand we will multiply the rational expression by $\frac{\sqrt{2}}{\sqrt{2}}$

$$\frac{3}{x\sqrt{2}} = \frac{3}{x\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$
$$= \frac{3\sqrt{2}}{x\sqrt{2^2}}$$

Step 2: Simplify

$$\frac{3}{x\sqrt{2}} = \frac{3\sqrt{2}}{x\sqrt{2^2}}$$
$$= \frac{3\sqrt{2}}{2x}$$

The next example will cover a situation where the index is greater than 2 and the radicand consists of more than just a single term.

Example 2: Simplify $\sqrt[3]{\dfrac{125y^2}{3x^2z}}$

Solution:

Step 1: Apply quotient rule for radicals

$$\sqrt[3]{\frac{125y^2}{3x^2z}} = \frac{\sqrt[3]{125y^2}}{\sqrt[3]{3x^2z}}$$

Step 2: Simplify the radical in the numerator

$$\sqrt[3]{\frac{125y^2}{3x^2z}} = \frac{\sqrt[3]{125y^2}}{\sqrt[3]{3x^2z}} = \frac{\sqrt[3]{5^3y^2}}{\sqrt[3]{3x^2z}} = \frac{5\sqrt[3]{y^2}}{\sqrt[3]{3x^2z}}$$

Step 3: Make the radicand in the denominator a perfect power

The index of the radical is 3 so our radicand needs to be a perfect cube. Right now we have one 3, two x's, and one z. So in order to have a perfect cube we need to multiply by the factor of 3^2xz^2 so that each term will be cubed.

$$\sqrt[3]{\frac{125y^2}{3x^2z}} = \frac{5\sqrt[3]{y^2}}{\sqrt[3]{3x^2z}} = \frac{5\sqrt[3]{y^2}}{\sqrt[3]{3^1x^2z^1}} \times \frac{\sqrt[3]{3^2xz^2}}{\sqrt[3]{3^2x^1z^2}} = \frac{5\sqrt[3]{y^2} \times \sqrt[3]{3^2xz^2}}{\sqrt[3]{3^3x^3z^3}}$$

Step 4: Multiply the radicals in the numerator and simplify the denominator

$$\sqrt[3]{\frac{125y^2}{3x^2z}} = \frac{5\sqrt[3]{y^2} \times \sqrt[3]{3^2xz^2}}{\sqrt[3]{3^3x^3z^3}} = \frac{5\sqrt[3]{9xy^2z^2}}{3xz}$$

The last situation that will be covered is where the denominator is a binomial term with a radical. The denominator must still be rationalized but now we must use a different technique in order to remove all radicals from the denominator. Since the denominator is a binomial term will must use its conjugate form when rationalizing the denominator. The conjugate form of the denominator is the binomial term that when multiplied by the denominator will result in the difference of two squares.

$$(a - b)(a + b) = a^2 - b^2$$

(a – b) and (a + b) are conjugates of each other

Example 3: Simplify $\frac{3}{1+\sqrt{2}}$

Solution

Step 1: Multiply the numerator and denominator by the conjugate

The denominator is $1+\sqrt{2}$ so the conjugate would be $1-\sqrt{2}$

$$\frac{3}{1+\sqrt{2}} = \frac{3}{1+\sqrt{2}} \times \frac{1-\sqrt{2}}{1-\sqrt{2}}$$

Step 2: Apply the distributive property to the numerator and denominator

$$\begin{aligned}\frac{3}{1+\sqrt{2}} &= \frac{3}{1+\sqrt{2}} \times \frac{1-\sqrt{2}}{1-\sqrt{2}} \\ &= \frac{3\left(1-\sqrt{2}\right)}{\left(1+\sqrt{2}\right)\left(1-\sqrt{2}\right)} \\ &= \frac{3(1)-3\left(\sqrt{2}\right)}{(1)^2-\left(\sqrt{2}\right)^2}\end{aligned}$$

Example 3 (Continued):

Step 3: Simplify

$$\frac{3}{1+\sqrt{2}} = \frac{3(1)-3\left(\sqrt{2}\right)}{(1)^2-\left(\sqrt{2}\right)^2}$$

$$= \frac{3-3\sqrt{2}}{1-2}$$

$$= \frac{3-3\sqrt{2}}{-1}$$

$$= -3+3\sqrt{2}$$