ROOTS AND RADICALS

This section expands the concept of exponents and how to discover the base of a number.

Recall that when a number is raised to a power it is written in the following format: b^x ; where b is the base and x is the power or exponent that the base is being raised to . In other words, it is the base being multiplied by itself x amount of times.

Example 1. $125 = 5^3 = 5*5*5$; $x^4 = x*x*x*x$

> **In this example 5 and x are known as roots. Since 5 is multiplied by itself 3 times it is said to be the cubed root (or third root) of 125. Since x is multiplied by itself 4 times it is said to be the fourth root of x^4 .**

The radical symbol $\left(\sqrt{\ }\right)$ is used to indicate that you are looking for a root, while the index number (n in this case) $\left[\sqrt[n]{b}\right]$ indicates what degree the base must be. The terms that will be used in these type of problems are:
n = index number , b = the radicand , $\sqrt{\ }$ = radical sign .

Example 2. $\sqrt[5]{32}$ **= ?**

> **This problem is looking for a base that when multiplied by itself 5 times yields 32. Since 32 = 2*2*2*2*2 the answer would be 2.**

The definition that governs these operations is : $\sqrt[n]{b} = |a|$ **if and only if $a^n = b$.**

There are two conditions that must always be observed:

1.) If n is an even positive integer:

a.) Every positive real number (b) will have two real n^th roots, one positive and one negative i.e. $\sqrt{9}$ = $|\pm 3|$ **= 3.**

b.) Negative real numbers (b) will not have real n^th roots i.e. $\sqrt{-9}$ **has no real solutions.**

2.) If n is an odd positive integer greater than 1 :

a.) All real numbers will have exactly one real n^th root.

b.) The real n^th root of a positive number is positive.

c.) The real n^th root of a negative number is negative.

Example 3. $\sqrt[3]{27}$ **= 3,** $\sqrt[3]{8}$ **= 2,** $\sqrt[3]{-125}$ **= -5**

Radical expressions may be simplified using the following rule:

$\sqrt[n]{ab} = \sqrt[n]{a} * \sqrt[n]{b}$ **; where** $\sqrt[n]{a}$ **and** $\sqrt[n]{b}$ **are real numbers**

Example 4.

$\sqrt[2]{36} = \sqrt[2]{4} * \sqrt[2]{9} = 2 * 3 = 6$, $\sqrt{18} = \sqrt{9} * \sqrt{2} = 3\sqrt{2}$

A rational expression beneath a radical , $\sqrt{\frac{x}{y}}$ **, may be resolved using the following rule:**

$\sqrt[n]{\frac{x}{y}} = \frac{\sqrt[n]{x}}{\sqrt[n]{y}}$ **; where** $\sqrt[n]{x}$ **and** $\sqrt[n]{y}$ **are real numbers and y** $\neq$ **0 .**

Keep in mind that a radical is said to be simplified if it meets these conditions:

1.) **No fraction appears with a radical sign ,** $\sqrt{\frac{x}{y}}$ **.**

2.) **No radical appears in the denominator of a fraction ,** $\frac{\sqrt[n]{x}}{\sqrt[n]{y}}$ **.**

3.) **When the expression is prime factored no radicand contains a factor raised to a power equal to or greater than the index number.**

Example 5. **Simplify** $\frac{3\sqrt{2}}{\sqrt{10}}$ **.**

Step 1. Resolve the denominator.

The first value that the $\sqrt{10}$ **can be multiplied by and get a perfect square as an answer is** $\sqrt{10}$ **. In order not to change the value of the problem we started with, the** $\sqrt{10}$ **must be put into a form that is equal to 1, as was done when multiplying values to get an LCD.**

$$\frac{3\sqrt{2}}{\sqrt{10}}\left(\frac{\sqrt{10}}{\sqrt{10}}\right) = \frac{3\sqrt{20}}{\sqrt{100}}$$

$$= \frac{3\sqrt{20}}{10}$$

Step 2. Simplify and reduce.

$$= \frac{3\sqrt{4}*\sqrt{5}}{10}$$

$$= \frac{3*2*\sqrt{5}}{2*5}$$

$$= \frac{3\sqrt{5}}{5}$$

This is the final answer because all 3 conditions discussed are met.