SOLVING RADICAL EQUATIONS

Radical equations are those equations that contain radicals with variables in the radicand. The rule needed to solve these equations is:

If $a = b$ then $a^n = b^n$

To resolve the radical, both sides of the equation must be raised to a power that is equal to the radicals index number.

Example 1. Solve the following.

a.) $\sqrt{3x+4}$ **= 8**

$\left(\sqrt{3x+4}\right)^2 = (8)^2$

3x + 4 = 64

3x + 4 – 4 = 64 – 4

3x = 60

$\frac{3x}{3} = \frac{60}{3}$

x = 20 This is the solution because the check works.

$\sqrt{3(20)+4} = 8$

$\sqrt{64} = 8$

8 = 8

Note: In cases where the index number of the radical to be resolved is even, it is critical that a check be performed to verify that the values you find are indeed the problems solutions. The next example demonstrates what may happen when working with this type of equation.

b.) $\sqrt{5q-1}$ **+ 3 = 0**

$\sqrt{5q-1}$ **+ 3 – 3 = 0 – 3**

$\sqrt{5q-1}$ **= -3**

$\left(\sqrt{5q-1}\right)^2 = (-3)^2$

5q – 1 = 9

5q – 1 + 1 = 9 + 1

5q = 10

$\frac{5q}{5} = \frac{10}{5}$

q = 2 This is *not* the solution according to the check.

$\sqrt{5(2)-1}$ **+ 3 = 0**

$\sqrt{10-1}$ **+ 3 = 0**

$\sqrt{9}$ **+ 3 = 0**

3 + 3 = 0

6 $\neq$ 0

c.) $\sqrt{m^2-4m+9}$ **= m - 1**

$\left(\sqrt{m^2-4m+9}\right)^2 = (m-1)^2$

$m^2-4m+9 = m^2-2m+1$

$m^2-4m+9 - m^2 + 2m - 1 = m^2 - 2m + 1 - m^2 + 2m - 1$

- 2m + 8 = 0

-2m + 8 + 2m = 0 + 2m

8 = 2m

$\frac{8}{2} = \frac{2m}{2}$

4 = m This is the solution because the check works.

$\sqrt{4^2-4(4)+9}$ **= 4 - 1**

$\sqrt{9}$ **= 3**

3 = 3

There are cases in which there are more than one radical term in the problem. The example that follows demonstrates some techniques in dealing with these problems.

Example 2. **Solve** $\sqrt{5m+6}$ **+** $\sqrt{3m+4}$ **= 2**

Step 1. **Select one of the radical terms and substitute a variable for it. In this case let** $\sqrt{3m+4}$ **= a.**

$\sqrt{5m+6}$ + $\sqrt{3m+4}$ = 2

$\sqrt{5m+6}$ + a = 2

Step 2. **Isolate the radical term.**

$\sqrt{5m+6}$ + a = 2

$\sqrt{5m+6}$ + a – a = 2 – a

$\sqrt{5m+6}$ = 2 – a

Step 3. **Square both sides of the equation.**

$\sqrt{5m+6}$ = 2 – a

$\left(\sqrt{5m+6}\right)^2 = (2-a)^2$

5m + 6 = 4 – 4a + a^2

Step 4. **Replace the substitution variable from step 1 with the original radical term. (Let a =** $\sqrt{3m+4}$ **)**

5m + 6 = 4 – 4a + a^2

5m + 6 = 4 – 4 ($\sqrt{3m+4}$ **) + (** $\sqrt{3m+4}$ **)^2**

5m + 6 = 4 – 4 ($\sqrt{3m+4}$ **) + 3m + 4**

5m + 6 = 8 – 4 ($\sqrt{3m+4}$ **) + 3m**

Step 5. Isolate the radical term.

$$5m + 6 = 8 - 4\left(\sqrt{3m+4}\right) + 3m$$

$$5m + 6 - 8 - 3m = -4\left(\sqrt{3m+4}\right)$$

$$2m - 2 = -4\left(\sqrt{3m+4}\right)$$

Step 6. Square both sides.

$$2m - 2 = -4\left(\sqrt{3m+4}\right)$$

$$(2m - 2)^2 = \left(-4\sqrt{3m+4}\right)^2$$

$$4m^2 - 8m + 4 = 16(3m + 4)$$

$$4m^2 - 8m + 4 = 48m + 64$$

Step 7. Set the equation to zero and solve for m.

$$4m^2 - 8m + 4 = 48m + 64$$

$$4m^2 - 8m + 4 - 48m - 64 = 48m + 64 - 48m - 64$$

$$4m^2 - 56m - 60 = 0$$

$$4(m^2 - 14m - 15) = 0$$

$$\frac{4(m^2 - 14m - 15)}{4} = \frac{0}{4}$$

$$m^2 - 14m - 15 = 0$$

$$(m - 15)(m + 1) = 0$$

$m - 15 = 0$ **or** $m + 1 = 0$

$m = 15$ **or** $m = -1$

Step 8. Check

$\sqrt{5m+6}$ + $\sqrt{3m+4}$ **= 2**	**or**	$\sqrt{5m+6}$ + $\sqrt{3m+4}$ **= 2**
Let m = 15		**Let m = -1**
$\sqrt{5(15)+6}$ + $\sqrt{3(15)+4}$ **= 2**	**or**	$\sqrt{5(-1)+6}$ + $\sqrt{3(-1)+4}$ **= 2**
$\sqrt{75+6}$ + $\sqrt{45+4}$ **= 2**	**or**	$\sqrt{-5+6}$ + $\sqrt{-3+4}$ **= 2**
$\sqrt{81}$ + $\sqrt{49}$ **= 2**	**or**	$\sqrt{1}$ + $\sqrt{1}$ **= 2**
9 + 7 = 2	**or**	**1 + 1 = 2**
16 $\neq$ **2**	**or**	**2 = 2**

Since when m equals 15 it does not yield a true solution but –1 does, the only solution for the problem is m = -1.